Wentworth · Key to Text-Book of Physics · 1898

KEY

TO

WENTWORTH & HILL'S

TEXT BOOK OF PHYSICS

BOSTON, U.S.A.:
GINN & COMPANY, PUBLISHERS
The Athenæum Press
[illegible]

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TYPOGRAPHY BY C. J. PETERS & SON,
BOSTON.

PREFACE.

THIS key is intended for teachers, and *for them only*. The publishers will take the utmost pains to keep the book from pupils ; and teachers are expected to exercise due care not to lose their copies, and not to leave them where pupils can have access to them.

Every teacher who obtains a copy must understand that he comes under an implied obligation not to let the book go out of his possession unless he sells it to the publishers.

It is hoped that many teachers who are pressed for time will find this edition of great advantage.

G. A. WENTWORTH.
G. A. HILL.

KEY TO PHYSICS.

CHAPTER I.

BALANCED FORCES.

Laboratory Exercises. Page 5.

1. *Show that air is compressible (as in Fig. 1, or otherwise).*

It can be readily shown that air is compressible by pushing a tumbler mouth downwards into water. The water rises in the tumbler. The volume of the air in the tumbler is diminished.

2. *Test rock salt or alum for porosity by putting a lump of the substance into water and slowly heating the water.*

Note whether the effect of heating is to cause bubbles of air to issue from the substance and rise to the surface of the water. If this happens, the substance must contain pores.

3. *Weigh a piece of chalk. Put it into water. Observe what takes place. After the chalk has been in the water some time, take it out and weigh it again. Compare the two weights. What inference as to the nature of chalk do you draw? Write in a notebook what you have done, what you have observed, and what you infer.*

The chalk will weigh more after it is taken from the water than it did before it was put into the water. The difference in weight, we infer, is due to the water that has soaked into the chalk; that is, we infer that the chalk is *porous*.

4. *Place a sponge over the hole on the bottom of a flower pot, and above the sponge some layers of sand and powdered charcoal. Then pour dirty water into the pot. What happens to the water? Write out an account of the experiment, as in Exercise 3, and explain the effect on the water. What property of matter is illustrated?*

After passing through the sand and the charcoal, the water is clear. This is another illustration of *porosity*. The water passes through the little openings between the grains of sand and the little lumps of charcoal, but these openings are too small to allow the particles of dirt to go through.

Class-room Exercises. Page 5.

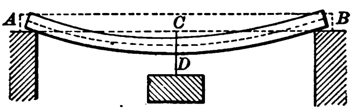
1. *What is matter? How do you know that air consists of matter?*

See § 1, page 1. We know that air is matter, because, when riding rapidly, we can feel its resistance. Air also occupies space.

2. *What property of matter is illustrated when you put a lump of sugar into a cup of tea and stir it around?*

Porosity. See Illustration 1, § 3, page 2.

3. *In the figure we see a beam bent by a heavy load. What effect has the bending on the length of the upper side? What effect has the bending on the lower side? How could you prove that these are the effects?*



The bending shortens the upper side of the beam, but lengthens its lower side. By measuring the length of the beam when unloaded, and then the length of its upper side and that of its under side when the beam is loaded, we shall find the statement made above to be true.

4. *A hollow globe of lead was once filled with water by Lord Bacon, and the hole securely closed. Then the globe was hammered so as to flatten it out and diminish the space inside. The outside was then found to be covered with a fine dew. What did this prove?*

This experiment proved that lead was porous.

5. *Water and alcohol when mixed contract in bulk. What do you infer from this fact?*

The inference is that the molecules of one of the liquids occupy spaces between the molecules of the other liquid.

6. *Is the falling of a stone a physical or a chemical change? the bending of a bow? the melting of ice? the boiling of water? the burning of coal? the rusting of iron? the ringing of a bell? the sprouting of a seed?*

The falling of a stone, the bending of a bow, the melting of ice, the boiling of water, and the ringing of a bell are physical changes. The burning of coal, the rusting of iron, and the sprouting of a seed are chemical changes.

7. *When phosphorus is burned in a closed vessel, it is found that the vessel and its contents weigh exactly the same after the burning as before. What truth does this experiment illustrate? What kind of a change does the phosphorus undergo?*

The indestructibility of matter. The phosphorus has undergone a chemical change.

Laboratory Exercises. Page 9.

1. *Measure the length of this page, the length of the printed part of the page, and the length of a full line of print on the page,*

(1) *Taking as the unit an inch;*

(2) *Taking as the unit a centimeter.*

Measure with an inch scale so that the error in your result may be less than $\frac{1}{32}$ of an inch; and with the centimeter scale so that the error may be less than 0.1 *of a centimeter.*

Record the various lengths in inches and in fractions of an inch, also in centimeters and fractions of a centimeter. Use measuring sticks on which each inch is divided into 32 parts, and each centimeter into millimeters.

2. *Measure with a centimeter scale the lengths of the sides of the triangle ABC (Fig.* 5). *Try to estimate the lengths correct to tenths of a millimeter.*

At first the student has difficulty in estimating the *tenths* of a millimeter, but practice brings confidence and accuracy.

3. *Find the area of the triangle ABC (Fig.* 5). *Area* $= \frac{1}{2}$ *base* $\times$ *altitude.*

The student should record the length of both the base and the altitude of the triangle.

4. *Find the area of the triangular piece of cardboard given you.*

It is a good plan to have several cardboard triangles of various shapes and sizes.

5. *Describe a circle with a pair of compasses. Then find its circumference and its area.*

Measure the radius or the diameter and then apply the formulas:

$$\text{Circumference} = \frac{44 \times \text{radius}}{7}; \text{ area} = \frac{22 \times (\text{radius})^2}{7}.$$

In the formulas, $\frac{22}{7}$ is an approximate value for π, but sufficiently accurate for use in this experiment.

6. *Find the surface and the volume of a rectangular block of wood.*

Measure its three dimensions, and find the product of the three numbers which express the dimensions in terms of the same unit. Each dimension should be measured four times, once along each of the four parallel edges which give the dimension in question, and the arithmetic mean of the results (their sum divided by 4) should be taken as the true value.

The object of making more than one observation of a physical quan-

tity is to eliminate as far as possible the errors of observation. By taking the mean of several observations we obtain a value which is much more likely to be very near the true value than that of any one actual observation.

A block of wood 3 in. long, 2.5 in. high, and 1.5 in. wide is convenient. It is a good exercise to get its volume not only in cubic centimeters, but also in cubic feet (a fraction of a cubic foot in this case).

7. *Find the surface and the volume of a cylinder.*

The area of the base may be found as in Ex. 5; or the circumference of the base may be found by rolling the cylinder carefully on paper until it has made just one revolution, and then the radius and area found by means of the formulas given in Ex. 5.

In case the circumference of the cylinder is to be found by rolling it on paper, the method of procedure should be to draw a straight line on the paper, then to lay the cylinder on its side so that the circumference of one of the ends shall be above the line. Mark the line and also the circumference at the point of contact. Roll the cylinder, so that its circumference always moves along the line, till the mark on the cylinder again comes in contact with the line. The distance between the marked points on the line will be the length of the circumference.

8. *Find the surface and the volume of a sphere.*

The surface and volume of the sphere are to be computed by means of the formulas

$$\text{Surface} = 4\,\pi r^2,\ \text{volume} = \frac{4\,\pi r^3}{3},\ \text{where } r = \text{the radius, and } \pi = \frac{22}{7}.$$

As the edges of the blocks, against which the measuring stick is placed, may be rounded by wear, it is a good plan to sight across the stick and along those sides of the blocks which are in contact with the sphere.

9. *Find by the displacement of water the volume of a small stone.*

The student should record the first level of the water, and also the level after the stone has been put into the jar. For small objects a cylindrical jar holding 100 ccm. is convenient.

10. *Find the weight of a given body.*

If a platform balance (Fig. 8) is used, weights up to one kilogram can be determined to 0.1 of a gram.

By means of a simple chemical balance, the weight of a body not exceeding 300 grams may be found to 0.01 of a gram.

In using the balance (Fig. 8, page 11), see that the rider is at the zero mark of the scale. Set the pans swinging. If the pointer does not make equal journeys to each side, put bits of paper in one of the pans

till the pointer's excursions become equal. Place the body to be weighed upon the left-hand pan and the weights upon the right-hand pan till, when swinging, the pointer again makes equal journeys from side to side. For making the final adjustments, in weighing a body, the rider is useful. Moving it towards the right is equivalent to adding weights to those already on the right-hand pan ; the exact amount thus added can be read from the horizontal bar along which the rider moves. As friction at the knife-edges of the balance is apt to affect the position of the pointer when the balance is at rest, weighings should always be taken when the balance is in motion and the pointer swinging ; time is saved and accuracy secured by this method.

11. *Find the density of the wood used in Exercise* 6.

See § 15, page 9. If the weight of the block in grams is divided by the number of cubic centimeters in the volume of the block, the result will be the density per cubic centimeter of the wood composing the block. If the weight of the block in pounds is divided by the number of cubic feet which the block contains, the result will be the density per cubic foot of the wood.

12. *Find the density of the stone used in Exercise* 9.

Find the weight of the stone in grams, and divide this by the number of cubic centimeters in the stone ; the quotient will be the density of the stone in the metric system.

13. *Find the density of a piece of wood, using for apparatus only a measuring glass, a metal sinker, and a balance with a set of weights.*

Find the weight of the wood in grams by means of the balance. Partly fill the measuring glass with water, and note the level at which the water stands. Put the sinker into the water, and again note the level at which the water stands. The difference of these two levels will give the volume of the sinker in cubic centimeters. Tie the sinker to the wood with a piece of fine thread, and by means of the measuring glass find the volume of the sinker and block together. From this volume subtract the volume of the sinker. The difference will be the volume of the wood in cubic centimeters. Divide the weight of the wood in grams by the number of cubic centimeters in the volume of the block. The quotient thus obtained will be the density of the wood.

Class-room Exercises. Page 11.

1. *What part of a kilometer is one millimeter ?*

1 mm. = 0.001 of a meter, and 1 meter = 0.001 of a kilometer. Therefore 1 mm. = 0.001 × 0.001 = 0.000001 of a kilometer.

2. *What is the difference in centimeters between* 4 *meters and* 8 *dm.?*

4 m. − 8 dm. = 400 cm. − 80 cm. = 320 cm.

3. *Reduce* 1 *square meter to square centimeters.*

1 qm. = $(100 \text{ cm.})^2$ = 10,000 qcm.

4. *Reduce* 16 *liters to cubic centimeters.*

1 liter = 1,000 ccm. Therefore 16 liters = 16,000 ccm.

5. *Reduce* 10 *cubic meters to liters.*

1 m. = 10 dm. ∴ 1 cbm. = 1,000 cdm. ∴ 10 cbm. = 10,000 cdm. Since 1 liter = 1 cubic decimeter, 10,000 cdm = 10,000 liters.

6. *Mt. Blanc is* 4,800 *meters high. Find the height in feet.*

1 m. = 3.28 ft. ∴ 4,800 m. = 4,800 × 3.28 = 15,744 ft.

Answer. Mt. Blanc is 15,744 ft. high.

7. *Which is the greater,* 8 *kilometers or* 5 *miles? Find the difference in feet.*

1 km. = 3,280 ft., nearly. ∴ 8 km. = 3,280 × 8 = 26,240 ft. And 5 miles = 5 × 5,280 = 26,400 ft. Hence 5 miles is greater than 8 km. by 160 ft., very nearly.

8. *Which is the greater,* 20 *centimeters or* 8 *inches? Express the difference in millimeters and tenths of a millimeter.*

20 cm. = 200 mm. 8 in. = 8 × 25.4 = 203.2 mm. Hence 8 in. is greater than 20 cm. by 3.2 mm.

9. *The Eiffel Tower in Paris is* 300 *meters high. Find the height of the tower in feet.*

300 × 3.28 = 984 ft.

10. *A body falls* 16.1 *feet in one second. What is this distance in centimeters?*

16.1 ft. = 193.2 in. = 193.2 × 2.54 = 490.728 cm.

11. *In London the average height of the barometer at the sea level is* 29.96 *inches. Find the height in millimeters.*

29.96 in. = 29.96 × 25.4 = 760.984 mm.

12. *How many square millimeters are there in* 1 *square meter?*

1 qm. = $(1{,}000 \text{ mm.})^2$ = 1,000,000 qmm.

13. *How many cubic centimeters are there in* 1 *liter?*

1 liter = 1 cdm. = 1,000 ccm.

14. *A certain distance is equal to* 89.6 *cm. Change the decimal point so that the number shall give the distance in meters.*

89.6 cm. = 0.896 m.

15. *What is the weight in grams of* 84 *ccm. of water? What is the weight in kilograms of* 84 *liters of water?*

1 ccm. of water weighs 1 gram ; hence, 84 ccm. weigh 84 g. 1 liter of water weighs 1 kg.; hence 84 liters weigh 84 kg.

16. *A gallon holds 4 quarts. Which is the larger, the quart or the liter? What is the difference in cubic centimeters?*

1 quart $= \frac{231}{4} \times 16.387 = 946.34925$ ccm. 1 liter $= 1{,}000$ ccm. Hence 1 liter is larger than the quart by 53.65 ccm., very nearly.

17. *What is the weight of 72 ccm. of lead, its density being 11.3 grams per cubic centimeter?*

$$72 \times 11.3 = 813.6 \text{ grams.}$$

18. *If a lump of sulphur weighs 400 grams and its density is 2.1 grams per cubic centimeter, what is the volume of the lump in cubic centimeters?*

Since 1 ccm. of sulphur weighs 2.1 grams, it will take as many cubic centimeters of sulphur to weigh 400 grams, as 2.1 is contained in 400, or 190.476 ccm.

19. *An iron rod is 8 meters long and 2 cm. in diameter. The density of the iron is 7.2 grams per cubic centimeter. What does the rod weigh?*

Weight of rod = volume × density $= 800\pi \times 7.2 = \frac{800 \times 22 \times 7.2}{7}$
$= 18{,}103$ grams, very nearly.

20. *A brass cylinder weighs 2.4 kilograms. Its height is 7 cm. and the radius of its base is 3.8 cm. Find the density of brass.*

Volume of cylinder $= 7 \times (3.8)^2 \times \frac{22}{7} = 317.68$ ccm.

Density of the brass $= 2400 \div 317.68 = 7.55$ grams per ccm.

21. *A glass ball weighs 600 grams and its diameter is 7.7 cm. Find the density of glass.*

Volume of sphere $= \frac{4}{3} \times \frac{22}{7} \times (3.85)^3 = 239.136$ ccm.

Density of glass $= 600 \div 239.136 = 2.509$ grams per ccm.

22. *What is the diameter of a platinum wire weighing 35 grams per meter? The density of platinum is 21 grams per cubic centimeter.*

Let x denote the diameter of the wire; then

$$35 = 100 \times \tfrac{22}{7} \times \frac{x^2}{4} \times 21.$$

whence $$x = 0.146 \text{ cm.}$$

23. *A straight piece of fine glass tubing weighs 1.2 gram. After a column of mercury 5 cm. long was drawn up the tube it weighed 2.2 grams. The density of mercury is 13.6 grams per cubic centimeter. Find the mean internal diameter of the tube.*

Weight of mercury $= 2.2 - 1.1 = 1$ gram.

Volume of cylinder of mercury $= 1 \div 13.6 = 0.07353$ ccm.

Area of base $= 0.07353 \div 5 = 0.0147$ qcm.

Diameter of tube $= \sqrt{\dfrac{0.0147 \times 4 \times 7}{22}} = 0.137$ cm.

24. *If the weight of* 1 *ccm. of a body in grams is denoted by* d, *and the weight of* 1 *cubic foot of water is* 1,000 *ounces, show that the weight of* 1 *cubic foot of the body is equal to* $\dfrac{1000\,d}{16}$ *pounds.*

Since 1 ccm. of water weighs 1 gram, the number d expresses how many times as heavy as water the body is when equal bulks are compared. Therefore

$$\begin{aligned}\text{Weight of 1 cubic foot of the body} &= d \times \text{weight of 1 cubic foot of water}\\ &= d \times 1000 \text{ oz.}\\ &= \frac{1000\,d}{16} \text{ lb.}\end{aligned}$$

See Text-book, p. 12.

25. *One cubic centimeter of marble weighs* 2.8 *grams. What is the weight of* 1 *cubic foot of marble in pounds?*

Since 1 ccm. of marble weighs 2.8 grams, marble is 2.8 times as heavy as water. One cubic foot of water weighs 1,000 oz., very nearly.

$\therefore$ 1 cubic foot of marble weighs 2,800 oz. or 175 lb.

26. *One cubic foot of granite weighs* 165 *lb. What is the weight of* 1 *ccm. of granite in grams?*

One cubic foot of granite weighs 16×165 or 2,640 oz. One cubic foot of water weighs 1,000 oz. $\therefore$ granite is 2.64 times as heavy as water. $\therefore$ 1 ccm. of granite weighs 2.64 grams.

27. *With a* 16-*lb. weight and a quantity of sand, how could you obtain a quantity of sand weighing exactly* 1 *lb.?*

Put the 16-lb. weight in one scale-pan, and weigh out just 16 lb. of sand. Remove the 16-lb. weight, and transfer sand from one scale-pan to the other till equilibrium is again obtained. Each pan will now contain 8 lb. of sand. Remove the sand from one of the pans, and transfer sand from the other pan to the empty one till each pan has 4 lb. of sand. Continue the process till each pan has only 1 lb. of sand.

Class-room Exercises. Page 17.

1. *Define force, and give examples of forces in action.*

See § 16.

2. *If a body is acted upon by a force and yet does not move, what is to be inferred? Give an example.*

That there is another force which balances the given force. See § 18.

3. *What kind of force is exerted when a man lifts his hand to his head? What other force is also acting?*

Muscular effort. The attraction of the earth.

4. *Name the forces which cause motion in the following cases: a waterfall, a revolving windmill, a swinging pendulum, a locomotive drawing a train of cars.*

The attraction of the earth causes the water to fall. The wind presses against the sails of the windmill and causes them to revolve. The earth's attraction causes the bob of the pendulum to descend as far as possible; then the bob ascends until the earth's attraction destroys the motion. The force of the steam exerted upon the pistons in the cylinders of a locomotive causes the driving wheels to revolve, and the friction between the driving wheels and the rails makes it possible for the locomotive to draw the train.

5. *A book is made to slide down a slate by raising one end of the slate. What force causes the motion? What force tends to stop the motion? See* § 35.

Gravity. Friction.

6. *If you push a sled on ice, it moves for a short time and then comes to rest. What force causes the motion? What force stops the motion?*

The muscular effort exerted. Friction.

7. *Is friction a force that tends to produce or to destroy motion?*

Friction tends to destroy motion.

8. *What are the two different effects of force?*

Change of motion and strain. See § 17.

9. *Give an example of a force so acting that*

(1) *It makes a body move faster and faster.*
(2) *It makes a body move slower and slower.*
(3) *It changes the direction of the motion.*

(1) When a body falls, gravity increases its speed.

(2) When a body is thrown upwards, gravity diminishes its speed.

(3) When a body is thrown into the air in an oblique direction, gravity changes the direction of the body's motion, and the body describes not a straight line, but a curve called a parabola.

10. *Give an example of a force so acting as to cause a strain. Is more than one force in action?*

If to the free end of a rubber cord attached to the edge of a table a weight is fastened, the cord will be strained by gravity pulling down on the weight. The table pulls up on the cord with a force equal to the downward pull of gravity.

11. *Give an example of two balanced forces; also one of three balanced forces.*

When a book rests upon a table the downward pull of gravity upon the book is balanced by the upward push of the table on the book.

An open door has three balanced forces acting upon it, a force at each of the hinges, and gravity.

The beam in Fig. 3 (page 5 of the Text Book) furnishes another example of three balanced forces.

12. *A bucket is lowered down a well at a uniform rate of speed. What two balanced forces are in action?*

Gravity, and the tension in the cord.

13. *A train is moving uniformly. What balanced forces are in action?*

The pull of the engine, the friction on the rails, and the resistance of the air.

14. *What kind of stress is exemplified when two railway trains come into collision? When a bucket of water is drawn up a well?*

Compression. Tension.

15. *Give an instance of a force so acting as to cause rupture. Is more than one force acting?*

If one end of a string is tied to a staple in the wall, and we pull hard enough at the other end of the string, the string will be ruptured, broken. The resistance of the wall is the other force acting.

16. *Point out two cases of action and reaction in Fig.* 9; *and two cases also in Fig.* 10.

The action of one hand and the reaction of the cord, also the action of the other hand and the reaction of the cord. The action of one jaw of the vise and the reaction of the wood; the action of the other jaw of the vise and the reaction of the wood.

17. *What forces constitute action and reaction in the following cases:*

(1) *A trunk lying on a floor.*
(2) *A man pushing against a wall.*
(3) *A horse drawing a cart at a uniform rate.*

(1) The weight of the trunk is the action; the resistance of the floor is the reaction.

(2) The push of the man is the action; the resistance of the wall is the reaction.

(3) The pull of the horse is the action; the resistance of the cart is the reaction.

18. *Describe how you would represent on paper a force of* 6 *lb. acting due east, and a force of* 8 *lb. acting on the same point due south.*

Choose some convenient length, as 1 inch, to represent 1 lb. To represent a force of 6 lb. acting due east, draw a line 6 inches long towards the right-hand edge of the sheet of paper, and put an arrow-tip on the line pointing in the direction in which the force acts. From the point whence the first line starts draw another line 8 inches long towards the bottom of the sheet of paper, and indicate its direction by an arrow-tip.

Laboratory Exercises. Page 23.

1. *Verify Hooke's Law by the method shown in Fig.* 12.

Hang the balance, a 30-lb. balance for example, by its ring to a firm support so that the balance-frame will not rest against any object. Prepare several equal masses of lead, each of which should weigh a pound or two. Test the equality of the masses, and therefore of the weights, by means of a good scale-pan balance. Hang one of these masses from the hook of the balance, and note the position of the pointer; then add another mass, and again note the position of the pointer. Do this till the pointer nearly reaches the lower limit of the balance-scale. In each case it will be noticed that the amount of stretching of the spring indicated by the pointer is proportional to the force applied ; that is, a force of 6 lb. will produce twice as much stretching as a force of 3 lb.

2. *Determine to what extent an India rubber cord obeys Hooke's Law.*

Fasten one end of a rubber cord about two feet long to a peg in the wall. Have a hook at the other end of the cord, and at some convenient position near the hook have a pointer attached to the cord. Then load the cord by adding small but equal masses one after another to the hook. Note the position of the pointer after each addition of mass. The pointer will indicate the amount of stretching which that portion of the cord between the peg and the pointer has undergone. It will be found that the cord will increase in length by unequal amounts as the load is increased by equal amounts. Twice the load will produce more than twice the amount of stretching.

3. *Verify Hooke's Law for bending stress* (§ 29), *and determine the elastic limit of the bar which you use.*

Support a rod, a meter stick for example, in a horizontal position by triangular prisms placed at right angles to the length of the rod near its ends. Place a third triangular prism, which should be a little larger

than the other two, parallel to the rod, and 5 cm. from its middle point. Place a light pointer over this prism and under the middle point of the rod. At 25 cm. from the prism put a vertical scale alongside of the pointer. Whatever the amount of bending of the rod may be, the pointer will magnify this amount five times. Suspend a small pan to the middle of the rod by means of a slender, yet strong, string. Read the position of the pointer on the scale, when no load is in the pan. Then load the pan with 100 grams, and note the reading of the pointer on the scale. Remove the load, and see whether the pointer returns to its original position. Then put 200 grams in the pan, and again read the position of the pointer. Remove this load, and again take the reading without load. Continue the experiment in this way, increasing the load by 100 grams each time, till several observations have been made. Do not, however, load the rod after its elastic limit has been reached. On examining the results, it will be found that the bending has been proportional to the load till the elastic limit was reached.

4. *Compare by experiment the forces required to break by bending stress two bars of pine wood alike in all respects except that one is twice as deep as the other.*

Support one of the bars on prisms as in the last experiment. Have the center of the bar over a hole in the table through which passes a stout string attached at one end to the center of the bar, and tied at the other end into the ring of a spring balance. Pull down gradually upon the hook of the spring balance till the rod breaks, and note the amount registered by the balance at the instant of breaking. Add to this the weight of the balance. Treat the other rod in the same way. The breaking strength of the thicker rod will be on the average about four times that of the thinner rod, or the breaking strengths are to each other as the squares of the thickness of each rod.

Class-room Exercises. Page 24.

1. *Explain, if you can, the reason for laws* (2) *and* (3) *in* § 26.

Law (2) is explained by considering that if a certain force will stretch by a certain amount a given length of wire, the same force would stretch an equal length of wire by a like amount, whether this wire is attached at one end to the wall or to the other piece of wire. In the latter case, we shall have the combined stretch of the two wires, or twice the stretch of one wire.

Law (3) is explained by considering that if the area of cross-section is doubled, a force that is applied will now have to stretch what is equiva-

lent to two wires placed side by side, consequently each of these wires having but half the force applied would be stretched only half as much as if the whole force were applied to a single wire.

2. *If a wire breaks under a weight of 12 kilograms, and 4 meters of the wire weigh 0.3 gram, how long must the wire be to break under its own weight when hung up by one end?*

One meter of the wire will weigh $\frac{3}{40}$ gram. It will take as many meters to weigh 12,000 grams as $\frac{3}{40}$ is contained times in 12,000, or $12,000 \div \frac{3}{40} = 160,000$ meters.

3. *If the safe pressure which brick can stand is 200 lb. per square inch, and brick is 2.2 times as heavy as water, how high can a brick wall be carried with safety?*

As a cubic foot of water weighs 62.5 lb., a column of water 1 foot high and 1 square inch in area of cross-section will weigh $\frac{62.5}{144}$ lb. A column of brick of the same dimensions will weigh $\frac{62.5}{144} \times 2.2$ lb. The wall can therefore be carried to a height equal to $200 \div \left(\frac{62.5}{144} \times 2.2\right) = 209.43$ feet.

4. *If a rod 40 ft. long is stretched 2 in. by a force of 200 lb., how much will this force stretch a similar rod 2 ft. long?*

As the strain varies directly as the length, a rod 2 ft. long will be stretched only $\frac{2}{40} = \frac{1}{20}$ as much as the rod 40 ft. long. $\frac{1}{20} \times 2 = 0.1$ in.

5. *A bar 8 qcm. cross-section is stretched 3 cm. by a certain force. What would be the elongation if the cross-section were only 1 qcm?*

As the strain varies inversely as the cross-section, the smaller bar will be stretched 8 times as much as the larger bar. $8 \times 3 = 24$ cm.

6. *If a weight of 1 ton bends a girder half an inch, how much would a weight of 10 tons bend the girder, if Hooke's Law holds true?*

As the bending is proportional to the load, 10 tons will bend the girder 5 in.

7. *If a beam 8 in. wide is bent 3 in. by a certain weight, how much would it be bent if it were 2 ft. wide?*

As the bending is inversely proportional to the width, a beam 2 ft. wide will be bent $\frac{1}{3}$ as much as a beam 8 in. wide, or 1 in.

8. *If a beam 12 ft. long is bent 2 in. by a certain weight, how much would it be bent if it were 6 ft. long?*

As the bending is proportional to the cube of the length, a beam 6 ft. long will be bent $(\frac{6}{12})^3 = \frac{1}{8}$ as much as the beam 12 ft. long, or $\frac{1}{4}$ in.

9. *If a beam* 10 *in. deep is bent* 1 *in. by a certain weight, how much would it be bent if the depth were* 5 *in.?*

As the bending is inversely proportional to the cube of the depth, a beam 5 in. deep will be bent $(\frac{10}{5})^3 = 8$ times as much as a beam 10 in. deep, or 8 in.

10. See Text-book, p. 24.

11. *If a pine beam* 5 *ft. long,* 2 *in. wide,* 3 *in. deep, is bent* 1 *in. by a weight of* 600 *lb., how much will a pine beam* 15 *ft. long,* 4 *in. wide,* 6 *in. deep, be bent by a load of* 2 *tons?*

Let x denote the deflection required.

The change from 600 lb. to 4,000 lb. makes the deflection $\frac{4000}{600}$ as much.

The change from 2 in. to 4 in. makes the deflection $\frac{2}{4}$ as much.

The change from 5 ft. to 15 ft. makes the deflection $\frac{15^3}{5^3}$ as much.

The change from 3 in. to 6 in. makes the deflection $\frac{3^3}{6^3}$ as much.

$$\text{Therefore, } x = 1 \times \frac{4000}{600} \times \frac{2}{4} \times \frac{15^3}{5^3} \times \frac{3^3}{6^3} = 11.25 \text{ in.}$$

12. *A pine floor beam is to be made* 20 *ft. long and* 8 *in. wide. What must be its depth to bend only* 2 *in. under a weight of* 12 *tons?* See Ex. 11.

Assuming, as in Ex. 11, that a pine beam 5 ft. long, 2 in. wide, 3 in. deep, is bent 1 in. by a load of 600 lb., and denoting the required depth of the beam by x, we have

$$2 = 1 \times \frac{24000}{600} \times \left(\frac{20}{5}\right)^3 \times \frac{2}{8} \times \left(\frac{3}{x}\right)^3$$

$$x^3 = 320 \times 27$$

$$x = 19.52 \text{ in., very nearly.}$$

Laboratory Exercises. Page 28.

1. *Verify the first two laws of friction given in* § 36.

If a spring balance is used, the pupil should correct the reading of the balance for the error when used in the horizontal position. The error arises from the fact that the weight of the hook and the rod connecting it to the spring do not of course pull on the spring when the balance is held horizontally. In some balances there is a wide pointer, the lower side of which indicates the reading for a horizontal position. See that the block moves at a uniform rate along the table.

Class-room Exercises. Page 28.

1. *Describe the use made of friction in docking a steamboat at a wharf, if you have seen the thing done.*

Hawsers are passed from the steamboat to the wharf, where they are fastened to strong posts. On the steamboat these hawsers are passed round cleats with one or two turns, and are slowly paid out. The friction at the cleats is very great, and it is this friction which quickly destroys the motion of the steamboat.

2. *Give examples of sliding and of rolling friction.*

Sliding friction is brought into action by the drag, the sled, and the piston of a pump. Rolling friction is brought into action by ball bearings, wheels, and hinges.

3. *Why is it harder for a horse to start a heavily loaded cart than to keep it in motion when once started?*

See law 4, § 36, page 25.

4. *The current of a river is less rapid near the banks than in the middle of the stream. Can you think of any reason for this?*

The friction between the banks and the water near them makes the water flow more slowly than in the middle of the stream.

5. *What is the coefficient of iron on iron if an iron cube weighing 200 lb. is made to slide along a horizontal iron rail by a force of 38 lb.?*

The extent of surface in sliding contact has no effect upon the force. Coefficient of friction $= \frac{38}{200} = 0.19$. See § 37.

6. *What force would be required to make the iron cube in Exercise 5 slide along the rail if a weight of 250 lb. were placed on top of the cube?*

$$\text{Force required} = 0.19 \times (200 + 250) = 85.5 \text{ lb.} \quad \text{See § 37.}$$

7. *What horizontal force is required to push a trunk weighing 300 lb. along a floor, the coefficient of friction being 0.35?*

$$\text{Force required} = 0.35 \times 300 = 105 \text{ lb.}$$

8. *What force is required to pull a sled along a smooth surface of ice, if the sled and its load weigh 300 lb. and the coefficient of friction is 0.05?*

$$\text{Force required} = 0.05 \times 300 = 15 \text{ lb.}$$

9. *If the coefficient of friction between a wooden beam and the floor is 0.4, and it requires a force of 140 lb. to make the beam slide along the floor, find the weight of the beam.*

Let x denote the weight; that is, the pressure of the beam upon the floor.

$$140 = 0.4 \times x, \quad x = 350 \text{ lb.}$$

10. *Show that law 2, § 36, is a necessary consequence of law 1.*

Law 1 is that friction varies directly as the pressure between the surfaces in contact; the meaning of law 2 is that the friction is independent of the extent of surface, provided the total pressure between the surfaces remains unchanged. When thus stated, law 2 evidently is implied by law 1. If, for example, the extent of surface is halved, it is true that the area over which friction is exerted is halved, but the pressure upon each unit of area is doubled; therefore the friction per unit of area is doubled, and the total amount of friction is unchanged.

Laboratory Exercises. Page 34.

1. *Verify by experiment the Parallelogram Law.*

The lines shown in Fig. 26 should be drawn with the utmost possible care.

2. *Find by the method suggested in § 47 the coefficient of friction between two smooth surfaces of dry pine wood.*

In measuring GH (see Fig. 25), take the distance between the table-top, on which the end of the board rests, and the *lower* side of the board. In measuring FG, take the distance from the point where the edge of the lower surface of the board rests on the table-top to the point G where a plumb-line from H would strike the table-top.

Class-room Exercises. Page 34.

1. *Forces of 6 lb. and 8 lb. act on a point. Find their resultant (1) if they act in the same direction, (2) if they act in opposite directions, (3) if they act at right angles.*

$$(1)\ 6 + 8 = 14 \text{ lb.};\ (2)\ 8 - 6 = 2 \text{ lb.}$$

$$(3)\ \sqrt{6^2 + 8^2} = \sqrt{100} = 10 \text{ lb.}$$

2. *Find the resultants of the following pairs of forces acting at right angles: (1) 3 lb. and 4 lb.; (2) 5 lb. and 12 lb.; (3) 8 lb. and 15 lb.*

$$(1)\ \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ lb.};\ (2) \sqrt{5^2 + 12^2} = \sqrt{169} = 13 \text{ lb.};$$

$$(3)\ \sqrt{8^2 + 15^2} = \sqrt{289} = 17 \text{ lb.}$$

3. *Two forces of 20 lb. each act on a point at right angles. Find their resultant. What angle does it form with each force?*

$$\sqrt{20^2 + 20^2} = \sqrt{800} = 20\sqrt{2} = 28.28 \text{ lb. An angle of } 45°.$$

If two equal forces act on a point, their resultant always bisects the angle between the forces. The parallelogram is a rhombus.

4. *The resultant of two concurrent forces acting at right angles is* 25 *lb. One of the forces is* 7 *lb. Find the other force.*

Let x denote required force.

$$\sqrt{7^2 + x^2} = 25 \text{ , } 49 + x^2 = 625.$$
$$x^2 = 576 \text{ , } x = 24 \text{ lb.}$$

5. *Resolve a force of* 16 *lb. into two perpendicular components, one of which shall be three times as large as the other.*

Let x denote one of the components; then $3x$ will denote the other component.

$$\sqrt{x^2 + 9x^2} = 16 \text{ , } 10x^2 = 256 \text{ , } x^2 = 25.6 \text{ , } x = 5.06 \text{ lb.}$$

Hence the components are 5.06 lb., and 15.18 lb.

6. *Resolve a force of* 60 *lb. into two perpendicular components, making equal angles with the given force.*

As the components are to make equal angles with the force, the components must be equal. Let x denote one of these components.

$$\sqrt{2x^2} = 60 \text{ , } x^2 = 1800.$$
$$x = 30\sqrt{2} = 42.42 \text{ lb.}$$

7. *Three concurrent forces have the values* 5, 7, 16. *How must they act if their resultant is* 4? 18? 28? 20? (*See* § 44.)

To have the resultant 4, 5 and 7 must act opposite to 16; to have the resultant 18, 7 and 16 must act opposite to 5; to have the resultant 28, all three forces must act in the same direction; to have the resultant 20, 5 and 7 must act in the same direction, perpendicular to 16.

8. *A* 100-*lb. weight is supported by two cords, each making an angle of* 30° *with the horizon. Find the tension of each cord.*

Since each cord makes an angle of 30° with the horizon, the cords make an angle of 120° with each other and with the direction of the 100-lb. weight. Hence, by (4) § 44, the tension in each cord is 100 lb.

9. *What force parallel to a smooth inclined plane* 40 *ft. long and* 24 *ft. high will support on the plane a body weighing* 1 *ton?*

By § 46, we have

$$\frac{P}{1} = \frac{24}{40}, \; P = 0.6 \text{ ton} = 1200 \text{ lb.}$$

10. *The base of a smooth, inclined plane is* 12 *ft. and its height* 5 *ft. What force acting parallel to the plane will support on it a weight of* 260 *lb.? What will be the pressure on the plane?*

$$\frac{P}{260} = \frac{5}{\sqrt{12^2 + 5^2}} = \frac{5}{13}; \therefore P = 100 \text{ lb.}$$

$$\frac{R}{260} = \frac{12}{\sqrt{12^2 + 5^2}} = \frac{12}{13}; \therefore R = 240 \text{ lb.}$$

NOTE. — In the language of engineers, the *grade* of an incline means the ratio of the height to the length of the incline, and is commonly expressed in the percentage form. Thus, a "four per cent grade" means that in walking 100 ft. up the incline, you rise 4 ft. Similarly, the *pitch* means the ratio of the height to the base.

11. *What force is needed to support* 1 *ton on a smooth incline, if the grade is* 8 *per cent?*

$$\frac{P}{1} = \frac{8}{100}; \therefore P = 0.08 \text{ ton} = 1600 \text{ lb.}$$

12. *It is desired to lay a track along rising ground so that (neglecting friction) a force of* 100 *lb. should just cause a weight of one ton to ascend the incline. What must the grade be?*

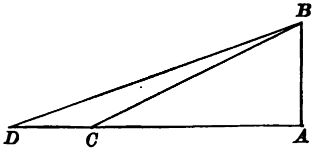
Let x denote the grade; then

$$x = \frac{100}{2000} = \frac{1}{20}. \quad \text{The grade must be } 5\%.$$

13. *The pitch of a plane is* 0.25, *and the coefficient of friction between a certain body and the plane is* 0.33. *If the body is placed on the plane, will it slide down or remain at rest?*

The body will remain at rest, since the pitch of the plane, in order that the body may slide down, must be 0.33, which is greater than 0.25. See § 47.

14. *Which is steeper, a* 3 *per cent grade or a* 3 *per cent pitch?*

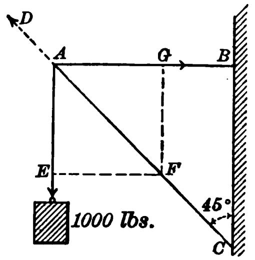


The 3 per cent grade is the steeper.

If, in the right triangle, ABC, $AB = 3$ and $BC = 100$, the base AC will be less than 100. If we produce AC to D, making $AD = 100$, and join BD, we have a right triangle, in which the pitch is 3 per cent, and the angle ACB is evidently greater than the angle ADB.

15. *A body weighing* 1,000 *lb. is supported in front of a wall by a tie-rod AB and a brace AC, as shown in Fig.* 27. *The angle $ACB = 45°$. Find the pull of the tie-rod and the push of the brace.*

The forces acting at A are 1,000 lb. downwards, the pull of the rod in the direction AB, and the push of the brace in the direction AD. Take AE to represent 1,000 lb. and complete the rectangle (here a square).



The push of the brace in the direction AD is just equal and opposite to the resultant of the weight, 1,000 lb., and the tension in the tie-rod. From the geometry of the figure, these two forces make equal angles with this resultant, and consequently they are equal; that is, the tension in the tie-rod equals 1,000 lb. The push of the brace $= \sqrt{1000^2 + 1000^2} = 1000\sqrt{2} = 1414$ lb.

16. *Three concurrent, balanced forces act, one toward the west, another toward the south, and the third toward the northeast. If the third force is* 6 *lb., find the other two.*

The resultant of the two unknown forces must bisect the angle between them, since it must act exactly opposite to the force directed towards the northeast. The two unknown forces must then be equal. Let x denote one of these forces.

$$\text{Hence } \sqrt{2x^2} = 6\ ,\ 2x^2 = 36\ ,\ x^2 = 18.$$
$$x = 3\sqrt{2} = 4.242 \text{ lb.}$$

17. *A cord attached to two pegs, A and B, fixed in the ceiling of a room, supports a ring weighing* 50 *lb. at C so that ACB is a right angle. Find the tension of the cord.*

The tensions are equal in the two segments of the cord, and their resultant must equal the weight of 50 lb.

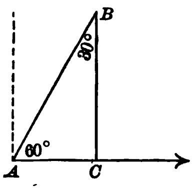
Let x denote the tension. Then

$$\sqrt{2x^2} = 50\ ,\ 2x^2 = 2{,}500\ ,\ x^2 = 1{,}250.$$
$$x = 25\sqrt{2} = 35.35 \text{ lb.}$$

18. *A car on a railway track is pulled by a horizontal force equal to* 100 *lb. in a direction making with the rails an angle of* 60°. *What is the force tending to make the car move forward?*

A part of this force of 100 lb. is expended in urging the car forward, the rest in pushing the car against the rails. We are called upon in this problem to resolve a force of 100 lb. into two forces at right angles to each other, one making an angle of 60°, and the other an angle of 30° with the given force.

Let AB represent the force of 100 lb., AC the direction of the motion. Draw BC perpendicular to AC. Then AC is the component which causes the motion. Now $AC = \frac{1}{2} AB = 50$ lb. (If one acute angle of a right triangle is half the other, the shorter leg is half the hypotenuse.)



Laboratory Exercises. Page 41.

1. *Verify the Law of the Lever by experiments like those shown in Figs.* 30 *and* 31, *p.* 37.

It is a good plan to ask the student to predict the force that must be applied at a given distance from the fulcrum in order to balance a given force applied at a given distance from the fulcrum.

2. *Verify the Law of Moments* (*p.* 40), *using a lever with at least two weights acting on each side of the fulcrum.*

In performing this exercise the weight of the lever should be eliminated by having the fulcrum at the center of gravity of the unloaded lever. Let three weights then be selected, and points of application for them also chosen. Then have the student compute where by the Law of Moments a fourth specified weight must act in order to produce equilibrium. The law is then verified by showing by experiment that the fourth weight applied at the computed point of application really does produce equilibrium.

Class-room Exercises. Page 41.

1. *What kind of lever is a pair of scissors? a pair of nut crackers? the treadle of a lathe? the oar of a boat? a pump handle? a claw hammer drawing out a nail? a pair of sugar tongs?*

First class; second class; second class; second class; first class; first class; third class.

2. *The arms of a lever of the first kind are* 9 *in. and* 25 *in. What weight acting on the longer arm will balance* 125 *lb. acting on the shorter arm? What is the pressure on the fulcrum?*

By the Law of the Lever, § 50, if x denotes the required force, we have

$$25x = 9 \times 125\ , \therefore x = 45 \text{ lb.}$$

The pressure on the fulcrum is $125 + 45 = 170$ lb.

3. *Weights of* 1 *lb. and* 12 *lb. balance on a lever. The longer arm is* 3 *ft. long. What is the length of the shorter arm?*

Let x denote the length of the short arm. Then we have, by § 50,

$$12x = 3 \times 1\ ; \therefore x = \tfrac{1}{4} \text{ ft.}$$

4. *An oar is* 12 *ft. long and the rowlock is* 2 *ft. from the handle. If the oarsman pulls with a force of* 100 *lb., what force is exerted at the rowlock in propelling forward the boat?*

The oar is a lever of the second class, the fulcrum being at the end of the oar which is in the water.

If x denotes the force exerted at the rowlock, we have

$$(12-2)\,x = 12 \times 100\,,\; x = 120 \text{ lb.}$$

5. *Find the mechanical advantage of the levers in Exs.* 2, 3, *and* 4.

By § 52, we have $\frac{25}{9}\,;\; \frac{3}{\frac{1}{4}} = 12\,;\; \frac{12}{10} = \frac{6}{5}\,.$

6. *Why is a piece of wire to be cut with shears placed near the rivet?*

In order to make the mechanical advantage as large as possible.

7. *A bar* 9 *ft. long supports at its ends weights of* 16 *lb. and* 20 *lb. Find the position of the fulcrum.*

Let $x =$ one arm; then $9 - x =$ the other arm.

$16\,x = 20\,(9-x)$; $x = 5$ ft. That is, the fulcrum must be 5 feet from the end supporting the 16-lb. weight.

8. *A horizontal rod, hinged at A, has two 8-lb. weights, acting* 12 *in. and* 18 *in. from A. What upward force* 16 *in. from A will produce equilibrium?*

In order to keep a body from turning about an axis the sum of the moments tending to produce rotation in one direction must equal the sum of the moments tending to produce rotation in the opposite direction. Therefore, if we denote by x the required force,

$$16\,x = 8 \times 12 + 8 \times 18 = 8 \times 30\;;\;\therefore\; x = 15 \text{ lb.}$$

9. *A stiff pole,* 12 *ft. long, sticks out horizontally from a vertical wall. It would break if* 20 *lb. were hung at the end. How far along the pole may a boy weighing* 80 *lb. venture with safety?*

Let x denote the distance which the boy would have to go along the pole in order that the moment of the boy's weight may be just equal to the moment of the weight of 20 lb. at the end of the pole. Then

$$80\,x = 20 \times 12\;;\therefore x = 3 \text{ ft.}$$

Hence the boy with safety may venture along the pole a distance a little less than 3 feet.

10. *A bar* 44 *in. long is in equilibrium with weights of* 5 *lb. and* 6 *lb. hung at the ends. Find the position of the fulcrum.*

Let x denote the distance of the fulcrum from the end from which the 6-lb. weight is hung; then the other arm of the lever will be $44 - x$; hence $6x = 5\,(44-x)\;;\therefore x = 20$ in.

11. *A lever of the second class, 10 ft. long, has to be applied so that a force of 20 lb. will overcome a resistance of 180 lb. What must be the distance from the resistance to the fulcrum?*

Let x denote the required distance.

Then $$180x = 20 \times 10 ;\ \therefore x = 1\tfrac{1}{9} \text{ ft.}$$

12. *Two men pull, each with a force of 36 lb., on oars 7 ft. long, the rowlocks being 2 ft. from the handle. Find the total force exerted on the boat.*

Let x denote the force exerted by each oar on the boat.

Then $$(7-2)x = 36 \times 7 ;\ \therefore x = 50.4 \text{ lb.}$$

Hence the total force exerted on the boat will be $50.4 \times 2 = 100.8$ lb.

13. *A horizontal bar, whose weight we neglect, 12 ft. long, rests on two props, A and B. A weight of 160 lb. is hung 3 ft. from A. Find the pressure on each prop.*

Treat the bar as a lever, having for a fulcrum first A and then B.

Regard A as a fulcrum, and let x denote the upthrust of the prop B upon the bar; this upthrust, by the law of action and reaction, is equal to the pressure of the bar on the prop. By the Law of the Lever, we have $$12x = 3 \times 160 ;\ \therefore x = 40 \text{ lb.}$$

That is, the pressure upon B is 40 lb.

Regard B as a fulcrum, and let y denote the upthrust of the prop A upon the bar; then $12y = 9 \times 160 ;\ \therefore y = 120$ lb.

That is, the pressure upon A is 120 lb.

14. *Two men,* A *and* B, *carry a load of 200 lb. on a pole between them. The men are 5 ft. apart, and the load is 2 ft. from* A. *What part of the load does each man bear?*

As in the preceding problem, regard the pole as a lever, having for a fulcrum first A's shoulder and then B's shoulder. Let x denote the upthrust of B's shoulder. We have

$$5x = 2 \times 200 ;\ \therefore x = 80 \text{ lb.}$$

Let y denote the upthrust of A's shoulder, B's shoulder being the fulcrum. We have $5y = 3 \times 200 ;\ \therefore y = 120$ lb.

A carries 120 lb., and B 80 lb.

15. *A rod 5 ft. long rests on two props. Weights of 4, 6, 8, and 10 lb. are hung from the rod at distances of 1, 2, 3, and 4 ft. respectively, from one prop. Find the pressure on each prop.*

Regarding the rod as a lever, and the prop from which the distances are measured as the fulcrum, we have, if x denotes the upthrust of the other prop,

$$5x = 1 \times 4 + 6 \times 2 + 3 \times 8 + 4 \times 10 = 80 ,\ \therefore x = 16 \text{ lb.}$$

Regarding the other prop as the fulcrum, we have, if y denotes the upthrust of the prop from which the distances are measured,

$$5y = 1 \times 10 + 2 \times 8 + 3 \times 6 + 4 \times 4 = 60, \therefore y = 12 \text{ lb.}$$

The pressure on one prop is 16 lb., on the other 12 lb.

NOTE. — Two parallel forces are said to be *like*, if they act in the same direction; *unlike*, if they act in opposite directions.

If P and Q denote two parallel forces, A and B their points of application, and R their resultant acting at C in the line AB, solve the following problems :

16. *P and Q like, $P = 4$ lb., $Q = 8$ lb., $AB = 9$ in.; find R and AC.*

In each of the examples 16, 17, 18, and 19, we shall have occasion to regard the resultant as reversed in direction so that it will balance the other two forces. The resultant thus reversed in direction is often called the *equilibrant.*

By § 53 (2), $\quad R = P + Q = 4 + 8 = 12$ lb.

Taking moments about A, we have, by § 53 (3),

$$12 \times AC = 8 \times 9 \text{ , } AC = 6 \text{ in.}$$

17. *P and Q like, $P = 7$ lb., $Q = 2$ lb., $AB = 3$ ft.; find R and AC.*

$$R = P + Q = 7 + 2 = 9 \text{ lb.}$$

Taking moments about A, we have

$$9 \times AC = 2 \times 3 \text{ , } AC = \tfrac{2}{3} \text{ ft.}$$

18. *P and Q unlike, $P = 4$ lb., $Q = 6$ lb., $AB = 1$ ft.; find R and AC.*

As P and Q are unlike, and Q is greater than P, Q must be the inner force; hence $\quad P + R = Q$, or $4 + R = 6$, $R = 2$ lb.

Taking moments about A, we have

$$2 \times AC = 6 \times 1 \text{ , } AC = 3 \text{ ft.}$$

19. *P and Q unlike, $P = 3$ lb., $Q = 9$ lb., $AB = 2$ ft.; find R and AC.*

Q is the middle force ; hence

$$P + R = Q \text{ , or } 3 + R = 9 \text{ , } R = 6 \text{ lb.}$$

Taking moments about A, we have

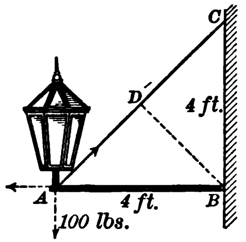
$$6 \times AC = 9 \times 2 \text{ , } AC = 3 \text{ ft.}$$

20. *A street lamp weighing 100 lb. is supported by a bracket projecting 4 ft. from a wall, as shown in Fig. 35. The tie-rod AC is attached to a point C in the wall 4 ft. above the brace AB. Find the pull along the tie-rod and the push along the brace.*

Let x = pull of tie-rod, y = push of brace.

Since there is equilibrium, the moment of x about $B =$ the moment of 100 lb. about B.

The arm of x is BD. By Geometry,



$$BD = DC,$$

and $\overline{BD}^2 + \overline{DC}^2 = \overline{BC}^2.$

$\therefore 2\,\overline{BD}^2 = 16$, whence $BD = 2\sqrt{2}$.

$\therefore 2\sqrt{2} \times x = 400$, whence $x = 141$ lb.

By taking the moments about C,

$4y = 100 \times 4$, whence $y = 100$ lb.

21. *Solve Ex. 20 by the Parallelogram Law.*

Let AC denote the pull of the tie-rod both in direction and magnitude. From the geometry of the figure the direction of the pull of the tie-rod bisects the angle between the two other forces; hence these forces must be equal; that is, the push of the brace is equal to 100 lb. The pull of the tie-rod is equal in magnitude to the resultant, x, of the two other forces; hence by the Parallelogram Law,

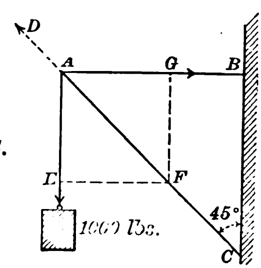
$$x^2 = 100^2 + 100^2 = 100\sqrt{2}\,;\ \therefore x = 141 \text{ lb.}$$

22. *Solve Ex.* 15, *p.* 35, *by the Law of Moments.*

In applying the Law of Moments in problems of this kind, it is well to choose the point, about which moments are to be taken, in such a way that the line of action of one of the unknown forces shall pass through this point; in this way the force will be eliminated.

Let T denote the tension of the tie-rod, and take moments about the point F. The push of the brace passes through F, and is thus eliminated. We have $T \times GF = 1000 \times EF$. But $GF = EF$. Therefore $T = 1000$ lb.

Let P denote the push of the brace, and take moments about G.



The arm of P

$=$ distance from G to the line AC.

$=$ half the diagonal of the square $AEFG$.

$$= \tfrac{1}{2}\,AG\sqrt{2}$$

$$P \times \tfrac{1}{2}\,AG\sqrt{2} = 1000 \times AG$$

$$P = \frac{2000}{\sqrt{2}} = 1000\sqrt{2} = 1414 \text{ lb.}$$

Laboratory Exercises. Page 49.

1. *With a wooden bar, a meter stick, and a set of weights, find by experiment the weight of the bar and the position of its C. G.* (*See Ex.* 8 *below.*)

Balance the wooden bar across the edge of the meter stick, and so find the center of gravity of the bar. With a pencil make a light mark on the bar where it balances. This mark gives the position of the center of gravity. Now place a weight, 100 grams for example, on the wooden bar near one end, and again adjust the wooden bar till it balances. Mark the point on the rod where it balances, thus recording the position of the fulcrum. Measure carefully from the fulcrum to the center of the weight ; suppose this distance is 25 cm. Measure also the distance from the fulcrum to the center of gravity of the stick; suppose this distance is 50 cm. Let x denote the weight of the stick; then, since the weight of a body acts just as if collected at the center of gravity of the body, we have $50x = 100 \times 25$, whence $x = 50$.

2. *Find* (1) *by experiment,* (2) *by construction* (*as explained in* § 58), *the C. G. of a triangular piece of cardboard.*

(1) Pierce the cardboard at one vertex with a needle, and enlarge the hole a little to make the cardboard swing freely when the needle, stuck in a horizontal position into the edge of a table, supports the cardboard. Make a little loop in one end of a piece of thread, and tie a bit of lead to the other end of the thread. Slip the loop over the needle, and carefully mark the point where this plumb-line crosses the lower edge of the cardboard. Dismount the cardboard, and join by a line the point where the needle pierced the cardboard and the marked point on the opposite edge. The center of gravity lies somewhere upon this line. Hang the cardboard by another vertex, and proceed as before. The new line thus obtained will also contain the center of gravity. Hence the required point will be at the intersection of the two lines.

(2) Turn the cardboard over, and draw two medians. Their point of intersection will be the center of gravity of the cardboard.

It is likely that the results of (1) and (2) will not precisely agree. Disagreement is brought about by the fact that the cardboard is not of uniform thickness, and that slight errors have been made in drawing the lines.

Class-room Exercises. Page 49.

1. *A ladder is raised from a horizontal to a vertical position by making it turn about one of its ends. Why does the force required to raise it become less as it approaches the vertical position?*

As the ladder approaches the vertical position, more and more of its weight is supported by the ground on which the end rests.

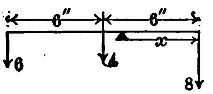
2. *Why is a wagon going along an uneven road more likely to overset if loaded with hay than if loaded with an equal weight of iron?*

When loaded with hay, the center of gravity is higher than when the wagon is loaded with iron. Consequently a displacement of the wagon loaded with hay will throw the center of gravity farther to one side than would be the case with the wagon loaded with iron.

3. *A uniform rod* 1 *ft. long and weighing* 4 *lb. has* 6-*lb. and* 8-*lb. weights fastened at its ends. At what point will it balance?*

As the rod is uniform, its center of gravity is at the middle of its length. At the center of gravity we may regard the weight of the rod concentrated.

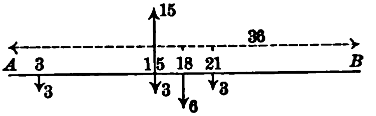
The fulcrum will evidently be between the middle of the rod and the end at which the 8-lb. weight acts. Let x denote the distance from this end to the fulcrum; then



$$8x = 6(12 - x) + 4(6 - x)$$
$$8x = 72 - 6x + 24 - 4x$$
$$18x = 96$$
$$x = 5\tfrac{1}{3} \text{ inches.}$$

4. *A uniform bar* 3 *ft. long and weighing* 6 *lb. has* 3 *rings, each weighing* 3 *lb., at distances* 3 *in.*, 15 *in., and* 21 *in. from one end. About what point of the bar will the system balance?*

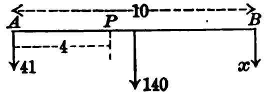
There will be a pressure of 15 lb. at the balancing point. Let x denote its distance from the end A, which has a ring 3 in. from it. Taking moments round A,



$$15x = 3 \times 3 + 15 \times 3 + 18 \times 6 + 21 \times 3$$

whence $x = 15$ in.

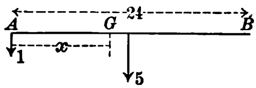
5. *A uniform beam* 10 *ft. long and weighing* 140 *lb. turns on a pivot* 4 *ft. from one end, where hangs a weight of* 41 *lb. What weight must be hung at the other end to maintain equilibrium?*



If x denote the required weight, and we take moments round the pivot P, we have
$$6x + 1 \times 140 = 4 \times 41$$
whence $x = 4$ lb.

6. *A uniform rod 2 ft. long and weighing 5 lb. has a weight of 1 lb. placed at one end. Find the position of the C. G. of the whole.*

The required position is that of the fulcrum on which the whole body will balance.



Let x denote the distance of the C. G. from A. Taking moments round G

$$x = (12 - x)\,5$$

whence $x = 10$ in.

7. *A uniform rod 14 ft. long balances at a point 4 ft. from one end, when 3 lb. is hung at that end. Find the weight of the rod.*

Let x denote the weight of the rod, and take moments round balancing point.

$$3\,x = 12\,,\ x = 4 \text{ lb.}$$

8. *A wooden bar AB, 40 in. long, balances on a pivot C 10 in. from A when a weight of 10 lb. hangs at A and 2 lb. at B. If the weight at A is changed to 4 lb., the bar balances at a point 15 in. from A. Find the weight of the bar and the position of the C. G.*

Let x denote the weight of the bar, y the distance of the *C. G.* from the end A. The pressure at the balancing point is $12 + x$ in the first case, and $6 + x$ in the second case. Taking moments round A in both cases, we obtain

$$\begin{aligned} xy + 80 &= 10\,(12 + x) \\ xy + 80 &= 15\,(\ 6 + x) \\ \hline 15\,(6 + x) &= 10\,(12 + x) \\ 3\,(6 + x) &= 2\,(12 + x) \\ x &= 6 \text{ lb.} \end{aligned}$$

By substitution in the first equation,

$$\begin{aligned} 6\,y + 80 &= 10 \times 18 \\ 6\,y &= 100 \\ y &= 16\tfrac{2}{3} \text{ in.} \end{aligned}$$

Review Exercises on Chapter I.

1. *Define the terms mass and weight so as to make clear the difference in their meanings.*

See §§ 11 and 12.

2. *Describe briefly the scale-pan balance and the spring balance. Why is a scale-pan balance unable to indicate variations in the weight of a body?*

See §§ 11, 14, 25.

3. *What is meant by the term stress? What is meant by the term strain? Give illustrations of the meanings of stress and strain. What*

is meant by tensile stress? What is meant by bending stress? What is meant by shearing stress?

See § 19. The pair of balanced forces which keep a rubber cord stretched form a stress, and the elongation produced is the strain. See §§ 26, 29, 32.

4. *If a body is acted upon by two or more forces, how can you tell whether the forces form a balanced system or not?*

Forces form a balanced system provided they cause no change in the state of the body as regards rest and motion.

5. *State and illustrate the law of action and reaction.*

See § 20.

6. *What are the elements of a force? How is a force represented?*

See § 21.

7. *State Hooke's Law. How is it illustrated in tensile stress? in bending stress?*

See §§ 25, 26, 29.

8. *Why is it better to make the depths of floor timbers greater than their widths?*

Because the stiffness of a beam increases as the cube of the depth, but increases only as the width.

9. *Mention some of the uses of elasticity.*

See § 34.

10. *Mention some of the uses of friction.*

See § 40.

11. *State the laws of sliding friction.*

See § 36.

12. *What is meant by saying that the coefficient of friction of pine on pine is* 0.25*? How much force will be required to make a pine plank weighing* 60 *lb. slide on a pine floor?*

The meaning is that the force of friction is one-fourth as great as the pressure between the surfaces. $0.25 \times 60 = 15$ lb.

13. *State the parallelogram law. If two concurrent forces act on a body, how can you find the magnitude and direction of a third force that will exactly balance the two given forces?*

See §§ 42, 43.

14. *Under what conditions will the resultant of two concurrent forces have the greatest value? Under what conditions will the resultant of two concurrent forces have the least value?*

When the two forces act in the same direction. When the two forces act in opposite directions.

15. *What is the law of equilibrium of a simple lever acted upon by two forces? Describe a lever such that a mechanical advantage equal to 2 is gained by its use.*

See § 50. One arm must be twice as long as the other.

16. *State the laws of equilibrium for three parallel forces.*

See § 53.

17. *State the law of moments.*

See § 54.

18. *Define the center of gravity of a body. How may it be found in the case of a triangle?*

See §§ 57, 58.

19. *Define and illustrate stable, unstable, and neutral equilibrium.*

See § 60.

20. *Upon what conditions does stability of equilibrium for a body resting on a plane depend?*

See § 61.

CHAPTER II.

FLUID PRESSURE.

Laboratory Exercises. Page 60.

1. *Verify the laws of liquid pressure stated on p. 54.*

The first law can be verified by keeping the center of the diaphragm, which is stretched across the mouth of the thistle tube, opposite some fixed point on the side of the jar, and making the diaphragm face in different directions. The index (that is, the short column of water in the glass tube) will remain at rest. This shows that there is no variation in pressure as the thistle tube is moved in the way described.

The second law can be roughly verified by lowering the thistle tube in the water. The deeper the thistle tube goes below the surface, the farther does the index move. This shows that the pressure increases.

The third law can be verified by moving the thistle tube to different positions in the liquid, but all at the same depth below the surface. For all these positions, the index shows no change, even when the distance below the surface is so great that the thistle tube can be moved beneath the piece of lead.

In all these tests it is necessary to be careful not to compress the tube. If the tube should be compressed, the index will either run out of the tube or it will fail to register properly. The water in the jar should be of the same temperature as the air in the room where the experiments are performed. It is a good plan to let the jar of water remain in the room several hours before the experiments are attempted.

2. *Show by the method suggested in Fig. 61 that the pressure upward at any point in a liquid is equal to the pressure downward.*

Hold the cover closely over the lower end of the cylinder by pulling on the string attached to the center.

Immerse the cylinder in the water, and push it down till the upward pressure of the water will hold the cover in place. Pour water into the cylinder till the cover falls off. If the cover had no weight, the water in the cylinder could be made to stand on a level with the water in the jar before the cover falls off.

Class-room Exercises. Page 60.

If the kind of liquid is not mentioned, assume it to be water.

1. *A vessel full of water is fitted with a tight cork. Why is it that a slight blow on the cork may suffice to break the vessel?*

The total pressure produced by the blow on the walls of the vessel will be as many times greater than the blow as the area of the inner surface of the vessel is times greater than the area of the end of the stopper in the vessel. The pressure thus exerted on the walls of the vessel is very great.

2. *In a hydraulic press the areas of the pistons are* 500 *sq. in. and* $\frac{1}{2}$ *sq. in. What pressure on the small piston will exert a pressure of* 1 *ton on the large piston?*

Let x denote the required pressure. Then, since the pressure upon any area is proportional to the area, we have

$$500 : \tfrac{1}{2} = 2{,}000 : x\text{, whence } x = 2 \text{ lb.}$$

3. *The diameters of the pistons in a hydraulic press are* 32 *in. and* 2 *in., respectively, and the pressure on the small piston is* 10 *lb. What pressure is exerted by the large piston?*

Let x denote the pressure exerted by the large piston.

The mechanical advantage of the press is $32^2 \div 2^2 = 256$.

Therefore $x = 10 \times 256 = 2{,}560$ lb.

5. *In a hydraulic press the radius of the press piston is* 3 *ft., and that of the pump piston is* 2 *in. The pump handle is* 5 *ft. long, and the piston is attached to the handle* 1 *in. from the fulcrum. What force must be applied to the pump handle to produce a pressure of* 243 *tons?*

Mechanical advantage of the lever is $60 \div 1 = 60$.

Mechanical advantage of the press alone is $36^2 \div 2^2 = 324$.

Therefore, if x denotes the required pressure,

$$x \times 60 \times 324 = 243 \times 2000\text{, whence } x = 25 \text{ lb.}$$

6. *Find the pressure (in kilograms per square centimeter)* 1 *km. below the surface of fresh water.*

A column of water 1 qcm. in area of cross-section, and 100,000 cm. long, weighs 100,000 grams, or 100 kilograms, which is the pressure required.

7. *A house is supplied with water from a reservoir* 241 *ft. above the ground. Find the pressure per sq. in. on a tap* 25 *ft. above the ground.*

The pressure is equal to the weight of a column of water $241 - 25 = 216$ ft. height, and 1 sq. in. in area of cross-section. The volume of water

in this column $= \frac{1}{144} \times 216 = 1.5$ cu. ft. Since 1 cu. ft. of water weighs 62.5 lb., the required pressure $= 1.5 \times 62.5 = 93.75$ lb.

8. *A trough 6 meters by 2 meters contains water 3 meters deep. Find the pressure on the bottom of the trough.*

Volume of water $= 6 \times 2 \times 3 = 36$ cbm.

Now 1 cbm. = 1,000 cdm. Since 1 cdm. of water weighs 1 kg., 1 cbm. of water will weigh 1,000 kg. Consequently the pressure on the bottom will be $36 \times 1{,}000 = 36{,}000$ kg.

9. *A mill dam is 40 ft. long, and the water is 15 ft. deep. Find the pressure upon the dam.*

Area of surface $= 40 \times 15 = 600$ sq. ft.
Mean depth $= \frac{15}{2}$ ft. Density = 62.5 lb. per cu. ft.
$\therefore$ total pressure $= 600 \times \frac{15}{2} \times 62.5 = 281{,}250$ lb.

10. *A trough 20 cm. long, 10 cm. wide, 8 cm. deep, is full of mercury. Find the pressure on its base and its sides if the density of mercury is 13.6 grams per cubic centimeter.*

Pressure on the base $= 20 \times 10 \times 8 \times 13.6 = 21{,}760$ grams.
Pressure on a narrow side $= 10 \times 8 \times 4 \times 13.6 = 4{,}352$ grams.
Pressure on a broad side $= 20 \times 8 \times 4 \times 13.6 = 8{,}704$ grams.
Total pressure on bottom and sides $= 21.760 + 2\,(4352 + 8704)$
$= 47{,}872$ grams.

11. *At what depth in water will the pressure on a horizontal square whose side is 50 cm. be 50 kg.?*

Let x denote the required depth in centimeters.

Then $2{,}500\,x = 50{,}000$, whence $x = 20$ cm.

12. *The lid of a vessel full of water is a square whose side is 20 cm. A pipe, leading from a side of the vessel, is filled with water to a height of 3 meters above the lid. How many kilograms must be placed on the lid to prevent the escape of the water?*

Let x denote the pressure of the water against the lid.

Area of lid = 400 qcm. Mean depth = 300 cm.

Therefore $x = 300 \times 400 = 12{,}000$ grams = 12 kg.

13. *A hole 6 in. square is made in a ship's bottom 20 ft. below the water line. What force is required to hold a piece of wood tightly over the hole?*

Area pressed $= 36$ sq. in. $= \frac{1}{4}$ sq. ft.
Depth $= 20$ ft.
Density $= 62.5$ lb. per cu. ft.
$\therefore$ force required $= 20 \times \frac{1}{4} \times 62.5 = 312.5$ lb.

14. *Find the pressure on an isosceles triangle in water, held vertically with its base in the surface, the base being* 50 *cm. and the height* 30 *cm.*

$$\text{Area of triangle} = \frac{50 \times 30}{2} = 750 \text{ qcm.}$$

Since the water presses on both sides of the triangle, the total area under pressure is 1,500 qcm.

$$\text{Mean depth} = \text{depth of C. G. of triangle} = \frac{30}{3} = 10 \text{ cm.}$$

$$\text{Pressure} = 1{,}500 \times 10 = 15{,}000 \text{ grams.}$$

15. *A cube whose edge is* 20 *cm. long is sunk till its top, which is horizontal, is* 60 *cm. below the surface. Find the pressure on one of its vertical sides.*

Area pressed $= 20^2 = 400$ qcm.
Mean depth $= 60 + 10 = 70$ cm.
Pressure $= 400 \times 70 = 28{,}000$ grams.

16. *A cubical box whose edge is* 8 *cm. is full of water, and a pipe opening into the side of the cube contains water to a height of* 48 *cm. above the bottom of the box. Find the pressure on the base, top, and one side of the box. Compare the pressure on the base and the weight of the water in the box, and explain why they are not equal.*

Each face of the cube $= 64$ qcm.
Mean depth of base $= 48$ cm.
Mean depth of top $= 40$ cm.
Mean depth of side $= 44$ cm.
Pressure on base $= 64 \times 48 = 3{,}072$ grams.
Pressure on top $= 64 \times 40 = 2{,}560$ grams.
Pressure on one side $= 64 \times 44 = 2{,}816$ grams.
Weight of the water $= (8)^3 = 512$ grams.

The pressure on the base is equal to the pressure of the top of the cube downwards (the equal and opposite reaction to the action of the water upwards) + the weight of the water.

$$3{,}072 = 2{,}560 + 512.$$

Laboratory Exercises. Page 68.

DETERMINATION OF SPECIFIC GRAVITY.

Solid heavier than water. In weighing the solid in water, take care to remove any air-bubbles that may be clinging to the solid ; see also that the solid is wholly covered by the water, and that it touches neither the sides nor the bottom of the vessel. Whenever it is necessary, as in

the present case, to attach the solid to the balance by a thread, use no more thread than is necessary, and have no ends left dangling in the water.

Solid lighter than water. Flotation Method. Measure the thickness of a rectangular block of wood at each of its four corners. Take the average of these four measurements. Put the block in water carefully, taking care to imprison no air-bubbles on its under surface. Sight carefully along the surface of the water, in the vessel in which the block is floating, and estimate as nearly as possible the exact place where the surface of the water would meet the block at each edge were it not for capillary action. Mark each of these points with a soft lead pencil. Remove the block from the water, and measure the depth, at each edge, to which the block sank. Take the average of these four measurements. By dividing the average depth to which the block sank by the average thickness of the block, we get the part of its total volume which was immersed. This is equal to the specific gravity of the wood.

Solid lighter than water. Sinker Method. In this experiment take care to brush off any air-bubbles, and see that the sinker is firmly fastened to the wood.

Solid soluble in water. In order that the ice may melt as little as possible while it is being weighed in kerosene, have the vessel containing the kerosene set into another somewhat larger vessel, and pack the space between the two vessels with broken ice. The temperature of the kerosene will thus, after a time, be reduced to the freezing point.

Liquids. First method. See that the bottle is perfectly dry when it is weighed empty. Take care that the bottle contains the same volume of the liquid as of water. This can be brought about by filling the bottle brim full of each liquid and then pushing the stopper into place, taking care that no air-bubbles are carried into the liquid by the stopper.

Liquids. Second Method. See that the piece of glass is dry when it is weighed in the air. Wipe the glass after it has been weighed in water, and before weighing it in the acid.

Other liquids, such as kerosene or a saturated solution of copper sulphate, can be used with advantage.

Class-Room Exercises. Page 69.

1. *What effect is produced on the weight of the vessel of water in Fig. 62 when the cylinder B is immersed in it? Explain.*

The weight is increased by an amount equal to the weight of a volume of water of the same size as *B*. This is explained by the fact that

when a body like B is submerged in water, it is pushed up by the water with a force equal to the weight of a volume of water of the same size as B. But action and reaction are equal and opposite, consequently B must react on the water with a downward force equal to the weight of the water displaced.

2. *Explain the use of life preservers.*

Life preservers are made of cork, which will float in water with about three-fourths of its volume above the surface. When attached to a man, the weight of the man that is unsupported by the water drags the life preserver down into the water till a weight of water is displaced equal to the unsupported weight of the man. The use made of the life preserver depends upon the fact that cork is light and bulky.

3. *When a ship passes from fresh water into salt water, will it sink deeper or rise higher? Why?*

It will rise higher. Salt water, volume for volume, is heavier than fresh water. Therefore the volume of salt water which must be displaced in order that its weight may be equal to that of the ship is less than when the water is fresh.

4. *Why does an iron steamship float when iron is nearly* 8 *times as heavy as water?*

Because the iron is not in a solid lump, but is made into thin sheets, which inclose so large a space that the vessel, after sinking till only a part of its bulk is below the surface, displaces its own weight of water, and consequently floats.

5. *Fishes are provided with an air-bag which they can inflate or contract at will. Show how this enables them to rise or sink in the water.*

When the air-bag is inflated, the volume of the fish is a little larger, the upthrust of the water on the fish is greater than before, and the fish rises. When the air-bag is compressed, the volume of the fish becomes a little smaller, the upthrust of the water is not so great now as it was before, and the fish sinks.

6. *Will lead (sp. gr.* 11.3) *sink or float in mercury (sp. gr.* 13.6)?

The lead will float, because, volume for volume, it is lighter than mercury.

7. *Find the weight of a cubic meter of petroleum (sp. gr.* 0.79).

A cubic meter of petroleum will weigh 0.79 as much as a cubic meter of water. 1 cbm. of water weighs 1,000 kg., therefore 1 cbm. of petroleum weighs $0.79 \times 1{,}000 = 790$ kg.

8. *Four cubic feet of cork weigh* 60 *lb. Find the sp. gr. of cork.*

$$\text{Specific gravity of cork} = \frac{60}{4 \times 62.5} = 0.24.$$

9. *What is the buoyant force on* 1 *cdm. of lead under water?*

One cdm. of lead will displace 1 cdm. of water. One cdm. of water weighs 1 kg. By the Law of Archimedes, the buoyant force is 1 kg.

10. *Find the volume of* 1 *kg. of cast iron (sp. gr.* 7.2). *What does the iron weigh under water?*

Let x denote the volume in cubic centimeters. Then $7.2\,x = 1{,}000$, $x = 138.889$ ccm. By the Law of Archimedes, the iron will weigh under water $1{,}000 - 138.889 = 861.111$ grams.

11. *If a piece of ivory weighs* 95 *grams in air and* 76 *grams in water, find its volume and its specific gravity.*

Weight of a volume of water equal to the ivory is $95 - 76 = 19$ grams. As 1 ccm. of water weighs 1 gram, the volume of the ivory is 19 ccm. Specific gravity of the ivory $= \frac{95}{19} = 5$.

12. *A piece of glass weighs* 24 *grams in air and* 16 *grams in water. Find its volume and its specific gravity.*

Weight of volume of water equal to that of glass is $24 - 16 = 8$ grams.

Volume of water equal to that of the glass	$= 8$ ccm.
Volume of the glass	$= 8$ ccm.
Specific gravity of the glass	$= 24 \div 8 = 3$.

13. *What will* 1 *cubic decimeter of stone (sp. gr.* 2.5) *weigh in water? If this stone is placed in mercury (sp. gr.* 13.6), *how much of it will project above the surface of the mercury?*

Weight of the stone	$= 2.5 \times 1 = 2.5$ kg.
Weight of the stone in water	$= 2.5 - 1 = 1.5$ kg.

Let x denote the number of cubic centimeters of the stone that will be above the surface of the mercury.

Then $$(1000 - x)\,13.6 = 2500.$$

$$13.6\,x = 11100\,, \text{ whence } x = 816 \text{ ccm., very nearly.}$$

Hence a little more than four-fifths of the stone will be above the surface of the mercury.

14. *A block of wood is placed in a vessel just full of water. It floats half submerged, and* 100 *ccm. of water run out. Find the weight, volume, and specific gravity of the wood.*

Since 100 ccm. of water run out, the weight of the block is 100 grams. As the block is half submerged, its volume $= 200$ ccm.

$$\text{Specific gravity of block} = \tfrac{100}{200} = 0.5.$$

15. *An iceberg has the form of a cube. Its height above the water is* 30 *ft. Find its entire height (sp. gr. of ice* 0.918, *that of sea water* 1.025).

If x denotes the height of the iceberg, x^3 will be the volume of the iceberg.

Weight of iceberg $= 0.918 \times 62.5 \times x^3$.

Weight of water displaced $= 1.025 \times 62.5 \times x^2 (x - 30)$.

Hence $$0.918 \times 62.5 \times x^3 = 1.025 \times 62.5 \times x^2 (x - 30),$$
$$0.918\, x = 1.025\, (x - 30),$$
$$0.107\, x = 30.75\,,\ x = 287.383 \text{ ft.}$$

The entire height of the iceberg is 287.383 ft.

16. *A piece of pomegranate wood (sp. gr. 1.35) is fastened to a block of pine (sp. gr. 0.65) of equal bulk. Will the two float or sink in water?*

Let x denote the volume of the piece of pomegranate wood.

Then $1.35\, x =$ the weight of the pomegranate wood,

$0.65\, x =$ the weight of the pine wood;

Weight of composite block is $1.35\, x + 0.65\, x = 2\, x$,

Weight of an equal volume of water $= 2\, x$.

Hence the composite block will sink in the water till none of it is above the surface. It has the same specific gravity as water.

17. *A cube of wood floating in water supports a weight of 200 grams. When the weight is removed the cube rises 2 cm. Find the size of the cube.*

Let x denote the length of one edge of the cube in centimeters.

Then $2\, x^2 =$ volume of block submerged by the weight of 200 grams.

Hence a volume of water equal to $2\, x^2$ cubic centimeters weighs 200 grams, or $2\, x^2 = 200$, whence $x = 10$.

18. *A barge with vertical sides, floating in fresh water, is 30 ft. long and 20 ft. wide. An elephant is driven upon the barge. When all is quiet, it is found that the barge has sunk just 4 in. Find the weight of the elephant.*

The weight of the elephant is equal to the weight of the water displaced, or $30 \times 20 \times \frac{1}{3} \times 62.5 = 12{,}500$ lb.

19. *A man weighs 75 kg. and his volume exclusive of his head is 72 cdm. How many cubic decimeters of cork (sp. gr. 0.25) are required to keep the man floating with his head above water?*

Let x denote the number of cubic decimeters of cork required; then the weight of the cork is $0.25\, x$.

Weight of floating body $= 75 + 0.25\, x$

Weight of water displaced $= 72 + x$

$$72 + x = 75 + 0.25\, x$$
$$0.75\, x = 3$$
$$x = 4$$

20. *A barge with vertical sides sinks to $\frac{1}{3}$ of its depth when unloaded, and to $\frac{3}{4}$ of its depth when loaded. If the barge weighs 4 tons, find the weight of the cargo.*

$\frac{3}{4} - \frac{1}{3} = \frac{5}{12}$. Since a weight of 4 tons causes the barge to sink to $\frac{1}{3}$ or $\frac{4}{12}$ of its depth, it will require a weight of 5 tons to make the barge sink $\frac{5}{12}$ of its depth more. Therefore the weight of the cargo is 5 tons.

21. *A lighter 20 ft. long, 8 ft. wide, 5 ft. deep, and weighing 2 tons, is filled with water, so that it is nearly sinking, and then fastened by chains to a wreck beneath. Find the lifting power exerted on the wreck when all the water is pumped out of the lighter.*

Let x denote the lifting power; then

$$x = \frac{20 \times 8 \times 5 \times 62.5}{2000} - 2 = \frac{50000}{2000} - 2 = 23 \text{ tons.}$$

22. *A cylinder floats in water with $4\frac{1}{2}$ in. of its length immersed. To what depth will it sink in a liquid whose specific gravity is 0.915?*

The depth to which the cylinder sinks in two different liquids varies inversely as the specific gravities of the liquids. Let x denote the required depth; then

$$0.915 : 1 = 4.5 : x, \text{ whence } x = 4.918 \text{ in.}$$

23. *A piece of glass weighs 47 grams in air, 22 grams in water, and 25.8 grams in alcohol. Find the specific gravity of alcohol.*

$$\text{Sp. Gr. of the alcohol} = \frac{47 - 25.8}{47 - 22} = 0.848.$$

24. *A body weighs 160 grams in air, 110 grams in water, and 80 grams in sulphuric acid. Find the specific gravity of the acid.*

$$\text{Sp. Gr. of the acid} = \frac{160 - 80}{160 - 110} = 1.6.$$

25. *A Fahrenheit's hydrometer, weighing 100 grams, requires 250 grams in the pan to sink it to the marked point in naphtha, and 350 grams to sink it to this point in water. Find the specific gravity of naphtha.*

$$\text{Sp. Gr. of the naphtha} = \frac{100 + 250}{100 + 350} = \frac{350}{450} = \frac{7}{9} = 0.778.$$

26. *A piece of copper sulphate weighs 3 oz. in air and 1.86 oz. in oil of turpentine (sp. gr. 0.88). Find the specific gravity of copper sulphate.*

$$\text{Weight of turpentine displaced} = 3 - 1.86 = 1.14 \text{ oz.}$$

$$\text{Weight of equal volume of water} = \frac{1.14}{0.88}$$

$$\text{Sp. Gr. of copper sulphate} = \frac{3}{\frac{1.14}{0.88}} = \frac{3 \times 0.88}{1.14} = 2.316.$$

27. *A piece of gold weighs* 9.7 *grams. A flask full of water weighs* 95 *grams. The gold is dropped into the flask, displacing some of the water. The flask and its contents now weigh* 104.2 *grams. Find the specific gravity of gold.*

$$\text{Sp. Gr. of gold} = \frac{9.7}{95 + 9.7 - 104.2} = \frac{9.7}{0.5} = 19.4.$$

28. *A piece of wood weighs* 120 *grams in air. A piece of lead weighs* 30 *grams in water. Both weigh* 20 *grams in water. Find the specific gravity of the wood.*

$$\text{Sp. Gr. of the wood} = \frac{120}{120 + 30 - 20} = \frac{120}{130} = 0.923.$$

29. *A solid weighs* 4 *lb. in air. A sinker weighs* 8 *lb. in water. Both weigh* 6 *lb. in water. Find the specific gravity of the solid.*

$$\text{Sp. Gr. of the solid} = \frac{4}{4 + 8 - 6} = \frac{4}{6} = 0.667.$$

30. *A solid weighs* 100 *grams in air and* 85 *grams in water. What will it weigh in a liquid whose specific gravity is* 0.8?

Weight of a volume of water equal to that of the solid $= 100 - 85 =$ 15 grams.

Weight of equal volume of liquid $= 0.8 \times 15 = 12$ grams.

Hence weight of solid in liquid $= 100 - 12 = 88$ grams.

31. *A nugget of gold and quartz weighs* 10 *oz. The specific gravity of gold is* 19.4, *that of quartz is* 2.1, *and that of the nugget is* 6.4. *Find the weight of the gold in the nugget.*

Let x denote the weight of the gold in the nugget.

Then $10 - x$ will denote the weight of the quartz in the nugget,

$\frac{x}{19.4}$ = weight of volume of water of the same size as that of the gold,

$\frac{10 - x}{2.1}$ = weight of volume of water of the same size as that of the quartz.

Hence
$$\frac{10}{\frac{x}{19.4} + \frac{10 - x}{2.1}} = 6.4$$

$$\frac{10 \times 19.4 \times 2.1}{2.1\,x + 194 - 19.4\,x} = 6.4$$

$$\frac{5 \times 9.7 \times 2.1}{194 - 17.3\,x} = 1.6$$

$$101.85 = 310.4 - 27.68\,x$$

$$27.68\,x = 208.55$$

$$\therefore x = 7.53 \text{ oz.}$$

Hence weight of gold in nugget is 7.53 oz.

32. *A body weighs* 18 *lb. and its specific gravity is* 3.5. *If the body is suspended by a string, find the tension of the string* (*a*) *when the body is immersed in water,* (*b*) *when it is immersed in a liquid whose specific gravity is* 2.

(*a*) The buoyant effect of the water is $\frac{18}{3.5}$ lb.; hence the tension of the string will be

$$18 - \frac{18}{3.5} = \left(1 - \frac{1}{3.5}\right) 18 = \frac{5}{7} \times 18 = 12.857 \text{ lb.}$$

(*b*) The buoyant effect of the liquid will be $2 \times \frac{18}{3.5}$ lb.; hence the tension of the string will be

$$\left(18 - 2 \times \frac{18}{3.5}\right) = 18\left(1 - \frac{2}{3.5}\right) = 18 \times \frac{3}{7} = 7.714 \text{ lb.}$$

33. *The specific gravity of pure milk is* 1.03. *The specific gravity of an article sold as pure milk is found to be* 1.02. *Prove that* 33 *per cent of this so-called milk is water.*

If 1 denotes the weight of the mixture, and x the weight of water, then $1 - x$ will denote the weight of milk.

Hence $$\frac{1}{\frac{x}{1} + \frac{1-x}{1.03}} = 1.02 \qquad \text{(See Ex. 31.)}$$

$$\frac{1.03}{1.03\,x + 1 - x} = 1.02$$

$$1.02 \times 0.03\,x = 1.03 - 1.02 = 0.01$$

$$x = \frac{0.01}{1.02 \times 0.03} = \frac{1}{3.06} = 0.33$$

Hence 33 per cent approximately of the mixture is water.

Laboratory Exercises. Page 75.

1. *Two Magdeburg hemispheres* (*Fig.* 72), *when fitted together, are easily separated while the air is within them, but when most of the air is removed great force is required to pull them apart.*

Take care that the edges of the cups, where they come together, are well oiled and free from dirt.

2. *When the air is removed from a wide-mouthed vessel over which a piece of bladder is fastened air-tight, at first the bladder is pushed inwards* (*Fig.* 73), *and finally it bursts with a loud report.*

Take care that the piece of bladder is very firmly fastened by the string wound round the neck of the vessel.

3. *Boil some water in a flask, and while the water is boiling, place over the mouth of the flask a ripe banana, end on, having first cut a few slits in the peel. Remove the lamp. Very soon you see the peel thrown aside in strips, while the banana enters the flask. Explain. Instead of a banana, a hard-boiled egg with the shell removed may be used.*

When the lamp is removed, the steam begins to cool and to condense. The pressure in the flask thus becomes less than the atmospheric pressure. The excess of the atmospheric pressure over the pressure in the flask forces the banana or the hard-boiled egg into the flask.

4. *Fill a glass brimful of water, cover carefully with a piece of cardboard, and then invert. The water does not run out. Explain.*

The water is supported in the vessel by the pressure of the air.

5. *Fill a pipette (Fig. 74) with water by suction. By keeping the finger pressed upon the upper end, the pipette can be carried about without the water escaping. By loosening the finger a little, the water can be made to escape by drops. Explain these phenomena. What is the explanation of suction?*

The pipette is used to remove a liquid from a vessel which we do not wish to disturb, or to put a liquid into a vessel drop by drop.

When the air is sucked out of a pipette, the lower end of which dips into water, the pressure of the atmosphere upon the surface of the water into which the pipette dips forces the water into the pipette. If the finger is pressed against the opening at the upper end of the pipette, no air can enter. The pressure of the atmosphere just balances the weight of the water column and the pressure of the air in the upper part of the tube. When, however, the finger is loosened a little, a small amount of air enters, and just enough water drops out as will make the pressure of the air now in the tube plus the weight of the water column precisely equal to the atmospheric pressure.

We often refer to the rise of a liquid in a tube as *suction*, when one end of the tube is placed in a liquid and the other end is connected with some means of removing the air. Of course it is the pressure of the air on the surface of the liquid in the vessel into which the tube dips that makes the liquid rise. If the pressure of the air did not get to the surface of the liquid, the liquid would not rise in the tube.

6. *Boil water in a closed flask provided with a safety tube A (Fig. 75), and conduct the steam through a glass tube into cold water at a lower level. Remove the source of heat. What happens? Explain. Repeat the experiment without a safety tube. What happens? Explain.*

When the source of heat is removed, bubbles of air enter the water

in the flask through the safety tube, and a column of water rises in the glass tube leading from the flask to a height about equal to the depth to which the safety tube reaches below the level of the water in the flask. The explanation of these phenomena is that the steam begins to condense after the source of heat has been removed, and the pressure of the atmosphere upon the surface of the water in the flask and in the safety tube, being greater than the pressure within the flask, causes the water to ascend in the tube leading from the flask, and the water to be depressed in the safety tube till bubbles of air finally pass into the flask through the safety tube.

When there is no safety tube, the pressure of the atmosphere forces the water from the flask through the tube and into the flask.

7. *Blow air into the bottle A through the tube B (Fig. 76). On removing the mouth, the water spouts out in a jet through B. When the jet subsides, place the bottle under the receiver of an air pump, and exhaust the air. The jet recommences. Explain.*

On blowing, the air passes through the tube and bubbles up through the water into the neck of the bottle, where it is compressed. When the mouth is removed, the pressure of this compressed air being greater than the atmospheric pressure, forces water up in a jet from the tube. When the bottle is put under the receiver of the air pump from which the air is exhausted, the jet recommences, because the confined air has a greater pressure than the rarefied air in the receiver.

8. *Take two small bottles, and fit one of them with a perforated rubber stopper. Fill this bottle nearly full of water, and connect the two bottles by a bent tube reaching nearly to the bottom of each one. Place the receiver of an air pump over the bottles, and exhaust the air. What happens? Readmit air. What happens? Explain.*

The water flows from the stoppered bottle into the other when the air is exhausted from the receiver, because the air, imprisoned in the stoppered bottle, has a greater pressure than the rarefied air in the receiver. When air is readmitted to the receiver, water flows from the open bottle into the stoppered bottle, because the air confined in the stoppered bottle has a less pressure than the air readmitted to the receiver.

Class-room Exercises. Page 77.

1. *Find the specific gravity of air referred to water. The density of air at 76 cm. and 0° C. is 1.293 gram per liter.*

One liter of water weighs 1,000 grams. Hence specific gravity of air

$$= 1.293 \div 1000 = 0.001293.$$

2. *Find the density at 76 cm. and 0° C. of hydrogen gas and of carbonic acid gas. Sp. gr. of hydrogen referred to air = 0.069; and that of carbonic acid gas = 1.529.*

Density of hydrogen = $0.069 \times 1.293 = 0.089217$ gram per liter.

Density of carbonic acid gas = $1.529 \times 1.293 = 1.976997$ gram per liter.

3. *If 1 liter of coal gas at 76 cm. and 0° C. weighs 0.78 gram, what is the specific gravity of coal gas referred to air?*

$$\text{Specific gravity of coal gas} = \frac{0.78}{1.293} = 0.603.$$

4. *Find the weight of the air in a room 6 meters long, 5 meters wide, and 4 meters high.*

Volume of room $= 6 \times 5 \times 4 = 120$ cbm.

Weight of air in room $= 120 \times 1000 \times 1.293 = 155{,}160$ grams.
$= 155.16$ kg.

5. *What is the pressure of the air in grams upon 1 qcm. when the barometer stands at 74 cm.?*

Pressure of the air $= 74 \times 13.6 = 1006.4$ grams per qcm.

6. *What is the pressure of the air in pounds upon 1 sq. in. when the barometer stands at 29 in.?*

$$\text{Pressure of the air} = \frac{29}{1728} \times 62.5 \times 13.6 = 14.27 \text{ lb. per sq. in.}$$

7. *If the bladder in Fig. 73 has a diameter of 20 cm., with what force is it pushed in when the pressure of the air below it has been reduced to 152 mm., the pressure above it being 760 mm.?*

$$\text{Force} = \frac{22}{7} \times 100 \times (76.0 - 15.2) \times 13.6 = 259878 \text{ grams}$$
$$= 260 \text{ kg., very nearly.}$$

8. *The piston of a steam engine has a diameter of 10 in., and the steam exerts a pressure upon it of 5 atmospheres. Find the effective force when the other side of the piston is exposed to the atmosphere.*

A pressure of 1 atmosphere means a pressure of about 14.7 lb. per sq. in. Therefore

Effective pressure per sq. in. $= 4 \times 14.7 = 58.8$ lb.

$$\text{Total effective pressure} = \frac{22}{7} \times 25 \times 58.8 = 4620 \text{ lb.}$$

9. *How high will a barometer filled with alcohol stand when a mercury barometer stands at 76 cm.? Sp. gr. of alcohol = 0.8; and that of mercury = 13.6.*

$$\text{Height of alcohol barometer} = \frac{13.6 \times 76}{0.8} = 17 \times 76 = 1292 \text{ cm.}$$

In general, the heights of barometric liquid columns vary inversely as the specific gravities of the liquids.

10. *During a storm a mercury barometer falls from* 30 *to* 29 *inches. Through what distance would a water barometer fall under the same conditions?*

As mercury is 13.6 times as heavy as an equal volume of water, a column of mercury 1 in. high would balance a column of water 13.6 in. high. Hence the water barometer would fall a distance of 13.6 in.

11. *Two glass tubes are arranged in a vertical position so that their lower ends dip into water and kerosene respectively, while their upper ends are connected to a mouth-piece. Air is then sucked out of the tubes till the height of the water column is* 260 *mm., and that of the kerosene column is* 329 *mm. Find the specific gravity of kerosene* (§ 71).

The heights of balancing columns of two liquids are inversely as their densities. Let x denote the specific gravity of kerosene ; then

$$329 : 260 = 1 : x \text{, whence } x = 0.79.$$

12. *If a barometer filled with glycerine reads* 325 *in. when a mercury barometer stands at* 30 *in., find the specific gravity of glycerine, and state the principles involved in the calculation.*

Each column is balanced by the atmospheric pressure. In this sense the two columns are balancing columns. Since the heights of balancing columns of two liquids are inversely as their densities, we have, denoting by x the specific gravity of glycerine,

$$325 : 30 = 13.6 : x \text{, whence } x = 1.255.$$

13. *A barometer tube, with a cross-section of 1 qcm., dips into a cistern of mercury whose cross-section, excluding the tube, is* 10 *qcm. If the mercury in the tube falls* 1 *cm., what is the real change in the pressure of the atmosphere?*

When the mercury in the tube falls, it enters the cistern and raises the level of the mercury there. As the area of the cistern excluding the tube is 10 qcm., a fall of 1 cm. in the tube will produce a rise of 0.1 cm. in the cistern. Hence the real change in the pressure of the atmosphere is $1 - 0.1 = 0.9$ cm.

14. *If the density of air, like that of water, were uniform and equal to that of air at the sea level, how high would the atmosphere extend? Assume the height of a water barometer to be* 34 *ft.*

Taking the specific gravity of air to be 0.001293, and letting x denote the height of the atmosphere, we have

$$0.001293 : 1 = 34 : x \text{, } x = 26295.43 \text{ ft., or nearly 5 miles.}$$

Laboratory Exercises. Page 84.

1. *Verify Boyle's Law by experiment.*

The method of performing the experiment is sufficiently described on page 84. In pouring mercury into the tube be careful that no air bubbles are carried round the bend by the mercury.

Class-Room Exercises. Page 84.

1. *A chemist generates a quantity of hydrogen gas which at* 0° *C. and under a pressure of* 95 *cm. measures* 800 *ccm. Find its volume at* 76 *cm.*

Let V denote the required volume. By Boyle's Law $P \times V = P' \times V'$, hence $76\ V = 95 \times 800$, whence $V = 1000$ ccm.

2. *A bladder holds* 30 *cubic inches of air under a pressure of* 15 *lb. per square inch. Find the size of the bladder if the pressure is reduced to* 5 *lb. to the square inch.*

Let V denote the required volume. Then

$$5\ V = 15 \times 30 \text{ , whence } V = 90 \text{ cu. in.}$$

3. *A bladder contains* 640 *ccm. of air under the pressure of* 76 *cm. At what pressure will the volume of the bladder be* 480 *ccm.?*

Let P denote the required pressure. Then

$$480\ P = 76 \times 640 \text{ , whence } P = 101.33 \text{ cm.}$$

4. *A bladder containing* 3 *cubic feet of air at atmospheric pressure is sunk to a depth of* 80 *ft. in water. To what bulk is the bladder compressed, the water barometer standing at* 32 *ft.?*

Let V denote the required volume. Then

$$(32 + 80)\ V = 32 \times 3 \text{ , whence } V = \tfrac{6}{7} \text{ cu. ft.}$$

5. *If the level of the mercury in the open arm of the manometer in Fig.* 86 *is* 57 *cm. higher than in the other arm, what is the pressure exerted by the gas in the vessel?*

The atmospheric pressure + the weight of 57 cm. of mercury.

6. *If the air in a compressed-air manometer is compressed to one-fifth of its volume under atmospheric pressure, what pressure does it exert?*

By Boyle's Law the volume of a gas varies inversely as the pressure. Hence, if the volume of the air becomes one-fifth of the volume occupied at atmospheric pressure, the pressure must be equal to 5 atmospheres.

7. *What will* 1 *liter of air weigh at* 0° *C., under a pressure of* 4 *atmospheres, if* 1 *liter of air at* 0° *C. and* 76 *cm. weighs* 1.29 *gram?*

Under a pressure of 4 atmospheres air becomes 4 times as dense, and therefore 4 times as heavy, as before; therefore

$$\text{Weight of the air} = 4 \times 1.29 = 5.16 \text{ grams.}$$

8. *What is the volume of* 1 *gram weight of air at* 76 *cm. and* 0° *C.? What is the volume at* 38 *cm. and* 0° *C.?*

If 1 liter of air at 0° C. and 76 cm. weighs 1.29 gram, 1 gram of air at 0° C. and 76 cm. will have a volume equal to $\frac{1}{1.29} \times 1000$ ccm., or 775.194 ccm.

If the pressure is reduced to 38 cm., the pressure is just halved; hence the volume will become twice as great, or $2 \times 775.194 = 1550.388$ ccm.

9. *A bubble of air rises to the surface from the bottom of a lake* 150 *meters deep. The volume of the bubble at the start is* 1 *ccm., and the water barometer at the time reads* 10 *meters. What is the volume of the bubble on reaching the surface?*

Let V denote the volume of the bubble when it reaches the surface.

Then $10\ V = (150 + 10) \times 1$, whence $V = 16$ ccm.

10. *At what depth in water would a bubble of air be compressed so as to become as dense as water, the height of the water barometer being taken as* 10 *meters and the density of air being* 1.29 *gram per liter?*

Let $x =$ required depth; then $x + 10$ represents in meters of water the pressure required; then apply the principle that density is proportional to pressure (§ 81).

The density of water = 1000 grams per liter.

$$10 : x + 10 = 1.29 : 1000$$
$$1.29\,x = 9987.1$$
$$\therefore x = 7741.938 \text{ meters.}$$

11. *A cylinder, open at the top, is inverted and immersed in water. Find the depth at which the cylinder will be half full of water; two-thirds full; three-fourths full.*

If the water barometer stands at the height of 34 ft., and if V denotes the volume of the cylinder, we will have, if x denotes the required depth,

$$(34 + x)\frac{V}{2} = 34 \times V, \text{ whence } x = 34 \text{ ft.}$$
$$(34 + x)\frac{V}{3} = 34 \times V, \text{ whence } x = 68 \text{ ft.}$$
$$(34 + x)\frac{V}{4} = 34 \times V, \text{ whence } x = 102 \text{ ft.}$$

12. *A cylindrical diving bell,* 15 *feet high, is sunk in water so that the top is* 51 *feet below the surface. Find how high the water will rise within the bell.*

If the water barometer stands at the height of 34 ft., and if x denotes the height to which the water will rise within the bell, we have

$$(15 - x)(34 + 51 + 15 - x) = 15 \times 34,$$
$$\text{or } x^2 - 115\,x + 990 = 0$$
$$x = \frac{115 \pm 96.24}{2} = 9.38 \text{ ft.}$$

NOTE. — In this solution the sides of the bell are assumed to be vertical; in this case the contained volume of air may be measured by its height. The smaller value of x must be chosen.

13. *A cylindrical diving bell,* 12 *feet high, is lowered until the water within the bell rises to a height of* 8 *ft. How far is the top of the bell below the surface of the water?*

If the water barometer stands at 34 ft., and x denotes the required distance, we have

$$4\,(34 + x + 4) = 12 \times 34$$
$$38 + x = 102$$
$$x = 64 \text{ ft.}$$

14. *If a small hole is made in the top of a diving bell under water, will water flow in or air flow out?*

The water at the top of the diving bell exerts less pressure than the water on the side of the diving bell. Now the air in the diving bell has the same pressure as the water outside on a level with the water within. Consequently the air will flow out.

15. *The standard balloon used in the siege of Paris,* 1870, *had a capacity of about* 70,000 *cubic feet. The density of air* $= 1\frac{1}{4}$ *oz. per cubic foot. Sp. gr. of coal gas (referred to air)* = 0.4 ; *and that of hydrogen* = 0.07. *Taking the weight of the balloon, car, etc., as* 1,000 *lb., find the lifting power* (1) *if coal gas is used,* (2) *if hydrogen gas is used.*

(1) Weight of air displaced by balloon $= \dfrac{70000 \times 5}{16 \times 4} = 5468.75$ lb.

Weight of coal gas $= 0.4 \times 5468.75.$

Lifting power $= 5468.75\,(1 - 0.4) - 1000 = 2281.25$ lb.

(2) Lifting power $= 5468.75\,(1 - 0.09) - 1000 = 3976.5625$ lb.

16. *The capacity of a balloon is* 30,000 *cubic feet. Weight of silk, car, etc.,* 560 *lb. Sp. gr. of the gas used* = 0.45. *Weight of* 1 *cubic foot of air* = 1.2 *oz. Find the lifting power of the balloon.*

Weight of air displaced by balloon $= \dfrac{30000 \times 6}{16 \times 5} = 2250$ lb.

Lifting power $= 2250\,(1 - 0.45) - 560 = 677.5$ lb.

17. *If you ascend* 10.5 *meters above the level of the sea, the barometer falls* 1 *mm. Why does it fall? If you rise* 10.5 *meters higher, will the barometer fall more or less than* 1 *mm.? Explain.*

Because the pressure of the air becomes less as you ascend.

If you ascend 10.5 m. higher, the barometer will fall less than 1 mm. The density of air diminishes as we ascend; and therefore we pass by less air in ascending the second 10.5 meters than in ascending the first 10.5 meters. Therefore the change in the height of the barometer is less.

Class-room Exercises. Page 92.

1. *Air is pumped out of a vessel till the mercury in a gauge connected with the vessel rises to a height of 68 cm. The barometer at the time reads 76 cm. What is the pressure of the air remaining in the receiver? What is its weight, if the air in the receiver originally weighed 19 grams?*

$$\text{Pressure of air in receiver} = 76 - 68 = 8 \text{ cm.}$$

The weight of a gas under constant volume varies directly as the pressure, therefore $76 : 8 = 19 : x$, whence $x = 2$ grams.

Hence the weight of the air remaining in the vessel is 2 grams.

2. *What must be the capacity of the barrel of an air pump in order that one half of all the air in a receiver holding 4 liters may be removed by a single stroke?*

The capacity of the pump barrel must be 4 liters; for then the 4 liters of air in the receiver will expand to 8 liters when the piston is raised, so that just one half of it will leave the receiver.

3. *If the receiver of an air pump is twice as large as the barrel, what fraction of the air in the receiver will be removed by each complete stroke of the piston?*

By § 84, one third of the air in the receiver at the beginning of any stroke will be removed by the complete stroke.

4. *If the volumes of the receiver and barrel of an air pump are 5 cubic feet and 1 cubic foot respectively, and the original pressure of the air is 30 in., what will the pressure be after 3 strokes?*

By § 84, we have
$$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times 30 = 17\tfrac{13}{36} \text{ in.}$$

5. *The barrel of a condenser has a capacity of 1 liter, and a hollow iron vessel has a capacity of 12 liters. How many strokes are required to raise the pressure of the air in the vessel to 10 atmospheres?*

12 full strokes will increase the pressure by one atmosphere. To increase the pressure to 10 atmospheres, there must be an increase of 9 atmospheres. This will require $9 \times 12 = 108$ complete strokes.

6. *The distance from the spout of a pump to the level of the water in the well is 20 ft. The area of the piston is 10 sq. in. Find the force (neglecting friction) required to raise the piston.*

Force required to raise piston $\frac{10}{144} \times 20 \times 62.5 = 86.806$ lb. See § 86.

7. *The area of the piston of a force pump is 15 sq. in., and the water is raised to a height of 64 ft. above the piston. Find the force required to push down the piston.*

The force will be equal to the weight of a column of water 64 ft. high and 15 sq. in. ($\frac{15}{144}$ sq. ft.) in area of cross-section, or $\frac{15}{144} \times 64 \times 62.5 = 416.66$ lb.

8. *How would the action of a siphon be affected by taking it to the top of a mountain?*

The atmospheric pressure at the top of the mountain is less than at the base; hence, the height of an obstacle over which a siphon could carry water at the top of the mountain would not be so great as at the base of the mountain.

9. *What would happen if a small hole were made in the short arm of a siphon in action? in the long arm?*

If a small hole were made in the short arm of the siphon, little air bubbles would enter and collect at the bend, and so stop the action. In case the hole was made in the long arm, the little bubbles would be swept out by the flowing liquid.

10. *What is the greatest depth a vessel can have in order that it may be emptied of water by means of a siphon?*

A depth not greater than the height of the water barometer.

11. *If a vessel 3 ft. deep is full of mercury, can it all be emptied by means of a siphon? Why?*

The vessel cannot be emptied completely by means of a siphon, because the pressure of the air will not support a column of mercury 3 ft. high.

12. *In what respect would the action of a siphon at the bottom of a caisson differ from its action in the free atmosphere?*

As the pressure of the air in a caisson is greater than in the open air, a deeper vessel could be emptied by the siphon in the caisson; moreover, the water would run out more rapidly.

13. *A vessel of water is under the receiver of an air pump, and a piece of wood is floating in the water. Will the wood rise higher or sink deeper if the air is removed from the receiver? Explain.*

The wood will sink a little deeper when the air in the receiver is removed. The wood is supported mostly by the buoyant effect of the

water, but also in part by the buoyant effect of the air. The buoyant effect of the water is equal to the weight of the water displaced by the immersed portion of the wood. The buoyant effect of the air is equal to the weight of the air displaced by that portion of the wood which lies above the water level. When the air in the receiver is removed, the buoyant effect of the air is taken away. Therefore the wood sinks till the increase in the buoyant effect of the water is just equal to the previous buoyant effect of the air.

14. *What effect on the wood in Ex.* 13 *would be produced if air were pumped into the receiver? Explain.*

The wood will rise a little higher, because the buoyant effect of the displaced air becomes greater.

Review Questions on Chapter II. Page 93.

1. *What is Pascal's Law? How is it applied in the hydraulic press?*
See §§ 65 and 66.

2. *Prove that the pressure exerted by a liquid at rest under gravity increases uniformly with the depth.*

See § 68.

3. *Describe a vessel so shaped that when filled with water the pressure upon the base shall be much greater than the weight of the water.*

See Fig. 54, § 70.

4. *What is the hydrostatic paradox? Explain it.*

See bottom of page 57.

5. *Give an example of unbalanced lateral pressure and its effect.*

See § 69.

6. *A mill dam has the shape of a rectangle. After a heavy rain the water rises to double its previous height, and the surface of the pond is also doubled. What change takes place in the pressure on the dam?*

The pressure on the dam is increased by the increase in the height of the water, not by the increase in the area of the pond. By the water rising to double the height the pressure on the dam is quadrupled, because the area pressed is doubled, and the mean pressure is also doubled.

7. *How is the total pressure against a mill dam found?*

$$\text{Total pressure (in lb.)} = \text{area (in sq. ft.)} \times \frac{\textit{height (in ft.)}}{2} \times 62.5.$$

8. *State the Law of Archimedes. How is it verified by experiment?*
See § 72.

9. *Give the general proof of the Law of Archimedes.*
See § 73.

10. *Apply the Law of Archimedes to a floating body.*

See § 74.

11. *How much weight will a cubic foot of any substance appear to lose when immersed in water?*

62.5 lb.

12. *Oil floats on water, but sinks in alcohol. Iron sinks in water, but floats on mercury. Explain these facts.*

Bulk for bulk oil is lighter than water, but heavier than alcohol; bulk for bulk iron is heavier than water, but lighter than mercury. By the Law of Archimedes, a body will float if bulk for bulk the body is lighter than the liquid.

13. *Define density and specific gravity, and point out the essential difference in their meaning. Why is the density of a substance in the metric system numerically equal to its specific gravity?*

See §§ 15 and 75.

14. *Describe a method of finding the sp. gr. of a solid that will sink.*

See § 75.

15. *Describe a method of finding the sp. gr. of a solid that will float.*

See § 75.

16. *Describe a method of finding the sp. gr. of a liquid.*

See § 75.

17. *In determining the specific gravity of a solid, the experimenter pays no attention to the fact that a large number of air bubbles were clinging to the solid when it was weighed under water. Will the result he obtains be too large or too small? Explain.*

The result will be too small. The presence of the air bubbles will make the weight of the solid in water appear less than it really is. Consequently the difference between the weight of the solid in air and the weight of the solid in water will be larger than it ought to be; and the quotient obtained by dividing the weight of the solid by the difference obtained will be too small.

18. *What is the weight of* 1 *liter of dry air under standard conditions? What are the standard conditions?*

1.293 gram. The standard conditions of temperature and pressure are 0° C. and 76 cm. They are the temperature and the pressure to which chemists and those who have to deal with gases refer the weights and volumes of gases for the sake of uniformity. See § 76.

19. *Describe Torricelli's experiment. What does it prove?*

See § 77.

20. *Describe a siphon barometer. What does it measure?*

See § 78. Barometers measure the pressure of the air.

21. *Describe the nature of the connection between the readings of a barometer and the state of the weather.*

See § 78.

22. *How will the reading of a barometer be affected if the tube is not in a vertical position?*

If the barometer is not vertical, the reading on the scale will be greater than it ought to be. The true height will be the vertical distance between the level of the mercury in the cistern and the top of the column of mercury in the tube.

23. *How do we ascertain the height of a mountain by a barometer?*

The barometer falls 1 inch nearly for every 900 ft. of rise into the atmosphere.

24. *If a barometer reads* 30 *in. at the sea level and* 20 *in. at the top of a mountain, will it read more or less than* 25 *in. at a point halfway up the mountain?*

It will read less than 25 in.; for the air decreases in density as we ascend, and therefore more air is passed by in ascending the lower half of the mountain than in ascending the upper half.

25. *A man tries to draw vinegar out of a cask, and finds that it will not run out. On removing the bung from the cask, the vinegar runs out freely. Explain.*

When the tap is opened, the weight of the vinegar tends to make the vinegar run out, but the pressure of the air will not permit this to happen. When, however, the bung is removed, the pressure of the air at the bung counteracts the pressure of the air at the tap, and the vinegar runs out, being free to obey the action of gravity.

26. *A circular piece of soft moist leather with a string attached to its center is called by boys a sucker. When the sucker is pressed down upon a flat-faced, heavy stone, it is found that the stone can be raised by pulling the string. Explain this phenomenon (Fig. 96).*

When the center of the piece of leather is pulled by the string from the stone, a vacuum is formed, and the pressure of the air holds the stone and the sucker together.

27. *State Boyle's Law, and illustrate its meaning by a numerical example.*

See § 79.

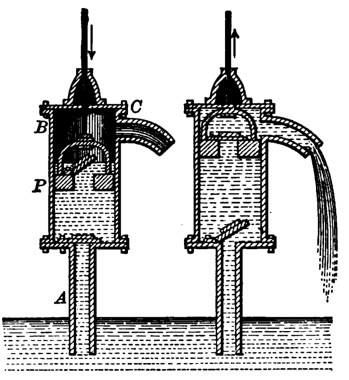
28. *If a diving bell is lowered to the same depth, first in fresh water, and then in salt water, which liquid will rise higher in the bell? In which case is the tension of the chain the greater?*

The salt water will rise higher than the fresh water, because the salt water is heavier and will compress the air most. The tension of the chain will be greater when the diving bell is in the fresh water. The buoyancy of the fresh water is less than that of the salt water for two reasons. First, the volume of water displaced by the bell is less, and secondly, the weight of the displaced volume per unit of volume is also less.

29. *Explain with a diagram the construction and action of an air pump.* See § 84.

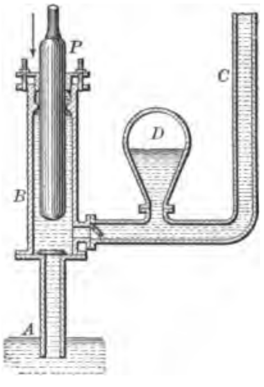
30. *Explain with a diagram the construction and action of a suction pump.*

Suppose that the pump (see figure) contains no water, and that the piston is pushed down. The air in the barrel *B* below the piston is thereby compressed till its pressure becomes so great that the piston valve is raised, and most of the air passes through the valve. Now let the piston be drawn up. The air below the piston is rarefied, its pressure diminishes, and the piston valve is closed by the superior pressure of the air above it. At the same time the pressure of the air in the pipe *A* raises the fixed valve, and air flows from the pipe *A* into the barrel *B*. The pressure of the air in the pipe *A* being thus lessened, the pressure of the atmosphere on the surface of the water in the well forces a column of water up some distance through the pipe *A*. By making another complete stroke with the piston, the water in *A* is raised still higher, and at last enters the barrel *B*. At the next down stroke of the piston some of the water will pass through the piston valve, and get above the piston. When the piston is raised, this water will be lifted to the spout *C*, and will then run out of the spout. From this time on, when the piston is pushed down, water will flow through the piston valve ; and when the piston is raised this water will be raised, and will run out of the spout ; more water at the same time being forced up into the barrel by the pressure of the atmosphere.



31. *Explain with a diagram the construction and action of a force pump.*

We will suppose that the piston has been raised and lowered till the water is in the pump barrel. When the piston is now pushed down (see figure), the water in the barrel is subjected to pressure. The consequence is that the valve opening to the supply pipe closes, the valve opening to the delivery pipe opens, and water is forced into the delivery pipe. When the piston is raised, the pressure of the water in the delivery pipe closes the valve opening to the barrel; and at the same time the pressure of the atmosphere on the surface of the water in the well forces more water through the supply pipe and the fixed valve into the barrel. During the up-stroke of the piston the ascent of water up the delivery tube would cease were it not for the elastic reaction of the compressed air in the air chamber *D*. This reaction makes the water continue to ascend the delivery tube, during the up-stroke, although its velocity is more or less diminished.



32. *Explain with a diagram the construction and action of a siphon.*

See § 88.

33. *Water is introduced into a small bottle to such an extent that the bottle will just float when inverted in water. When the bottle is pushed down some distance in the water, it will sink to the bottom and remain there. Explain (Fig. 97).*

As the bottle is pushed deeper into the water, the air contained in the bottle is compressed. Consequently the volume of water displaced by the bottle and the air it contains becomes less. When the volume of displaced water becomes so small that the weight of the displaced water is less than that of the bottle and the contained air, the bottle will sink.

34. *A lump of lead and a large wooden ball balance each other when suspended from the pans of a balance. A receiver is set over the balance, and the air is exhausted. The wooden ball descends. Explain.*

Air, as well as water, exerts a buoyant effect on bodies. This buoyant effect is equal to the weight of the air displaced. Hence the large wooden ball will be buoyed up by a greater force than that which is

exerted upon the small lump of lead. If the air is removed, the wooden ball will be deprived of a greater supporting force than the lump of lead; therefore the wooden ball descends.

35. *Explain the action of the pneumatic inkstand (Fig. 98).*

When so much ink has been removed from the mouth of the inkstand as to bring the level in the mouth below the dotted line in Fig. 98 (p.94), a bubble of air will ascend through the ink in the body of the inkstand to the air at the top. The pressure of this air is thereby increased, and forces some ink from the body of the inkstand into the mouth. This action will continue until the level of the ink in the inkstand falls nearly to the dotted line.

36. *Explain the action of a blacksmith's bellows.*

The action of a simple bellows is illustrated in Figs. 1 and 2. When

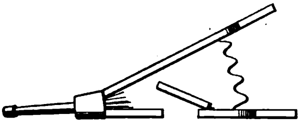


FIG. 1.

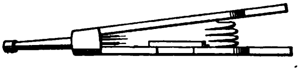


FIG. 2.

the handles of the bellows are pulled apart, as in Fig. 1, the air within is rarefied, and the exterior air forces the valve open and rushes in. When the handles of the bellows are pushed together, the air within is compressed, the valve closes, and the air rushes out in a stream through the nozzle.

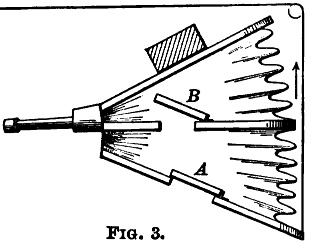


FIG. 3.

The action of a blacksmith's bellows will be apparent by examining Fig. 3. When the smith pulls the rope, air is driven from the lower part *A* to the upper part *B*. When the smith stops pulling the rope the weight which rests on top of the bellows compresses the air in *B*, closes the valve, and drives a stream of air through the nozzle.

CHAPTER III.

HEAT.

Class-room Exercises. Page 104.

1. *What would happen to the compound bar in Fig.* 104 *if it were cooled?*

The bar would bend so as to bring the zinc on the concave side.

2. *Why is a glass stopper often loosened by pouring hot water on the neck of the bottle?*

The heat from the water makes the neck of the bottle expand, thus increasing the size of the mouth.

3. *The longer a pendulum is the slower it vibrates. Hence, show why a clock loses time in summer and gains in winter.*

In winter the clock gains time because the pendulum becomes shorter by contraction, and consequently vibrates more rapidly. In summer the clock loses time because the pendulum becomes longer by expansion, and consequently vibrates more slowly.

4. *A compensation pendulum is a pendulum so made that its length shall remain constant in spite of changes of temperature. The principle of action in Graham's mercury pendulum is illustrated in Fig.* 110. *Observe that the effect of expansion upon the mercury in the vessel at the end of the pendulum is to raise its center of gravity a little. Hence, explain how the compensation is effected.*

When Graham's mercury pendulum has its temperature raised, the mercury expands; this tends to raise the center of oscillation, and so increase the rate of vibration; at the same time, however, the rod which supports the vessel of mercury is lengthened. By properly adjusting the length of the rod and the quantity of mercury, these effects may be made to neutralize each other. When the pendulum is cooled, the contraction of the rod tends to raise the center of oscillation, while the contraction, and consequent sinking of the mercury in the cup, tends to lower the center of oscillation. Again, these two tendencies neutralize each other, and the rate of the clock suffers no change.

5. *What temperature C. is equivalent to 68° F.?*

$$C. = \frac{5}{9}(68 - 32) = 20°.$$

6. *What temperature F. is equivalent to 15° C.?*

$$F. = \frac{9}{5} \times 15 + 32 = 59°.$$

7. *Change 0° F. to the equivalent Centigrade reading.*

$$C. = \frac{5}{9}(0 - 32) = -17°.78.$$

8. *Change —40° F. to the equivalent Centigrade reading.*

$$C. = \frac{5}{9}(-40 - 32) = -40°.$$

9. *An iron steam pipe is 60 ft. long at 0° C. How long does it become when steam at 110° passes through it? Value of k,* 0.000012.

$$\text{Length} = 60 + 60 \times 110 \times 0.000012 = 60.0792 \text{ ft.}$$

10. *The distance between two marks on a brass bar is 1 meter at 20°. What is the distance at 80°? Value of k,* 0.000017.

$$\text{Distance} = 100 + 100 \times (80 - 20) \times 0.000017 = 100.102 \text{ cm.}$$

NOTE. — The full method of solving this question would be, first, to find the distance at 0°, and then at 80°. But k is so small that for all practical purposes the change in distance between 20° and 80° may be taken as equal to $100 \times 0.000017 \times (80 - 20) = 0.102$ cm.

11. *A steam pipe is made in sections each 15 ft. long. Assuming that the variation in temperature ranges from 10° C. to 110° C., find how much play must be allowed at each joint.*

$$\text{Allowance} = 15 \times (110 - 10) \times 0.000012 = 0.018 \text{ ft.} = 0.216 \text{ in.}$$

12. *The iron rails of a railway are each 30 ft. long. What space must be left between two consecutive rails to allow for expansion for a range of temperature of 50° C.?*

$$\text{Space} = 30 \times 50 \times 0.000012 = 0.018 \text{ ft.} = 0.216 \text{ in.}$$

13. *Describe the behavior of water if heated from 0° C. to 100° C.*

The water will contract till it attains a temperature of 4° C., then it will expand till it reaches 100° C.

14. *Two thermometers, one filled with mercury and the other with water, are marked so as to agree at the two fixed points; but their readings are found to differ at temperatures between the fixed points. Explain.*

Mercury expands all the way between the two fixed points at a rate which is very nearly constant. Water contracts slightly when heated from 0° to 4°, and then expands from 4° to 100°, but at a rate which increases with the temperature. The consequence is that the readings on the two scales will not agree between the two fixed points. The reading on the water thermometer will be less than that on the mercury thermometer. The difference of the readings will gradually diminish as the temperature approaches the boiling point.

15. *Compare water, mercury, and air as thermometric substances.*

Water is not a good thermometric substance on account of its irregularity in expansion. Mercury is much more uniform in its rate of expansion than water.

Air expands much more rapidly, and its rate of expansion is uniform throughout the scale. See § 97.

Water is useless for measuring temperature except between 0° and 100°; moreover, water is much less sensitive to heat than either mercury or air.

Mercury freezes at $-39°$ C., and boils at 350° C.; therefore mercury cannot be used for measuring temperatures outside these limits. Air can be used for measuring any temperature, and when great accuracy is desired, is the best of all thermometric substances. But much time and considerable skill are required in order to make a good determination of temperature with an air thermometer. For most purposes mercury is the best substance that can be used.

16. *Find the weight of* 1 *liter of boiling water, the coefficient of expansion of water being* 0.00046.

At 4° C. 1 liter of water weighs 1 kg. 1 kg. of boiling water will have a volume of $1 + 1 \times (100 - 4) \times 0.00046 = 1.04416$ liters

$\therefore$ 1 liter of boiling water will weigh $\dfrac{1}{1.04416}$ kg., or 0.958 kg.

17. *The sp. gr. of mercury at* 0° *is* 13.6. *What is its sp. gr. at* 200°? *Coefficient of real expansion of mercury* = 0.000182.

If 1 ccm. of mercury at 0° is heated to 200°, its volume becomes $1 + 200 \times 0.000182 = 1.0364$ ccm. Its weight is 13.6 grams.

Therefore the specific gravity of mercury at $200° = \dfrac{13.6}{1.0364} = 13.12.$

18. *To what temperature must a gas be heated at constant pressure in order that its volume may be twice what it is at* 0°?

By § 98, we have $1 : 2 = 0 + 273 : x + 273$

$$273 + x = 546$$
$$\therefore x = 273°.$$

Or by § 97, since a gas increases by $\frac{1}{273}$ of its volume at 0° for every degree increase of temperature, its volume at zero will be doubled at 273°.

19. *A certain quantity of gas measures* 360 *ccm. at* 30° *C. What will be its volume at* 0° *C.?*

Let x denote its volume at 0°.

Then $30 + 273 : 0 + 273 = 360 : x$

$$303\,x = 273 \times 360$$
$$101\,x = 91 \times 360 = 32760$$
$$x = 324.35 \text{ ccm.}$$

20. *A glass vessel full of air at* 0° *under atmospheric pressure is gradually heated. If the vessel can safely stand a pressure of* 4 *atmospheres, to what temperature mdy it be heated?*

Since the pressure varies directly as the absolute temperature (when the volume is constant), we have

$$0 + 273 : x + 273 = 1 : 4$$
$$x + 273 = 1092$$
$$\therefore x = 819°.$$

21. *An open glass flask holds* 1 *liter of air weighing* 1.293 *gram at* 0°. *What weight of air will it contain at* 100°?

Let x denote the volume which the air will occupy at 100°.

Then
$$0 + 273 : 100 + 273 = 1 : x$$
$$273\,x = 373$$
$$x = 1.366 \text{ liter.}$$

At 100° this air still weighs 1.293 gram; hence at 100°, 1 liter will weigh $\frac{1.293}{1.366} = 0.947$ gram.

22. *A chemist generates* 69 *liters of gas at* 10° *C. and a pressure of* 70 *cm. Find the volume of the gas at* 0° *C. and* 76 *cm.*

We have
$$\frac{VP}{T} = \frac{V'P'}{T'},$$

hence
$$\frac{60 \times 70}{10 + 273} = \frac{V' \times 76}{0 + 273}$$
$$283 \times 76 \times V' = 60 \times 70 \times 273$$
$$21508\,V' = 1146600$$
$$\therefore V' = 53.31 \text{ ccm.}$$

23. *The pressure on a gas is doubled, and the temperature raised from* 25° *to* 116° *C. How is the volume altered?*

Let a denote the volume of the gas before the pressure is doubled. Then, when the pressure is doubled, the volume will be $\frac{a}{2}$.

If x denotes the volume when the temperature is altered, we have

$$25 + 273 : 116 + 273 = \frac{a}{2} : x,$$
$$298\,x = 389 \times \frac{a}{2}$$
$$596\,x = 389\,a$$
$$\therefore x = 0.6527\,a.$$

That is, the volume becomes 0.6527 of its original volume.

24. *If the volume of a gas is doubled, what change in the temperature is required in order that the pressure may remain unaltered?*

Doubling the volume of the gas makes its pressure only one-half as great. Now, as the pressure for a given volume of gas varies directly as the absolute temperature, the absolute temperature must be doubled in order that the pressure of the gas may remain the same as it was before any change was made in its volume.

If the original temperature of the gas is t° C. the required temperature is $2\ t^\circ + 546^\circ - 273^\circ = 2\ t^\circ + 273^\circ$.

Laboratory Exercises. Page 113.

1. *Determine the melting point of wax, and try to ascertain whether wax contracts or expands when it melts.*

The wax can be melted in a beaker, and the temperature at which the wax melts can be found with a thermometer, as illustrated in Fig. 111. When the wax cools, its surface will not be level, but slightly concave. This shows that the wax contracts when it changes from the liquid to the solid state.

A more delicate way of finding the melting point of wax is to fill with melted wax a tube of small bore. The wax quickly hardens in the tube. Close one end of the tube in the flame of a lamp. Bind the tube to the bulb and stem of a thermometer, and plunge them into warm water. Gradually warm the water till the wax melts. Note the temperature. Gradually cool the water till the wax hardens. Note the temperature. The average of these two temperatures, which should be very nearly the same, is the melting point of the specimen of wax which you have used.

2. *Demonstrate the effect of freezing on water by filling a test tube with water, and surrounding it by a mixture of salt and snow.*

Have plenty of ice and salt round the test tube. The water increases in bulk at the instant of solidification.

3. *Perform Tyndall's experiment of passing a wire through ice (p.* 108).

With a large cake of ice, weights of 20 lb. or more should be used.

4. *Perform Franklin's experiment described on p.* 111.

A 250 ccm. flask, with thick sides and a rubber stopper, is well adapted for performing this experiment. A Florence flask sometimes breaks.

Class-room Exercises. Page 114.

1. *State facts illustrating the force with which water expands when it freezes.*

See § 100. Water pitchers and water pipes are cracked frequently when the water in them freezes.

2. *A cannon is filled with water and the mouth tightly sealed up. What will happen* (1) *if the cannon is surrounded by a freezing mixture?* (2) *if the cannon is surrounded by fire?*

(1) The cannon would be likely to be fractured by the expansion of the water when it freezes. (2) The cannon would explode with great violence under the enormous pressure of the water vapor which would form within the cannon.

3. *Why is iron a good metal for making castings? Why are coins stamped and not cast?*

Iron expands on solidification, and thus takes a sharp impression of the mould. Gold, silver, and copper, of which coins are made, contract on solidification, and consequently cannot take a sharp impression of the mould. Hence coins must be stamped with dies.

4. *Explain the adhesion of two pieces of ice when they have been pressed together and the pressure is removed.*

See § 101.

5. *A piece of ice at 0° C. is heated under atmospheric pressure to 120° C. Trace the changes in volume which occur.*

The ice melts with a diminution of bulk. The water continues to diminish in bulk till the temperature of 4° C. is reached; then the water expands till it reaches 100° C., when it is changed into steam with an enormous increase in volume. As the steam is heated from 100° to 120°, it continues to expand if free to do so.

6. *Is heat absorbed or set free in the process of evaporation? Illustrate your answer by an experiment.*

Heat is absorbed. Surround the bulb of a thermometer with cotton. Saturate this cotton with alcohol, or better, with ether. As the liquid evaporates, the thermometer falls. Heat has been taken from the bulb and its contents by the liquid during the process of evaporation.

7. *Why is a windy day the best for drying clothes?*

Because the layers of air, in contact with the clothing, which have taken moisture from the clothing, are rapidly carried away, and replaced by layers of dryer air.

8. *Why is it impossible to cook certain vegetables in an open vessel on the top of a high mountain? How can they be cooked there?*

Because when water boils on the top of a mountain its temperature is not so high as when boiling at the sea level. By putting the vegetables into a closed vessel, however, the vegetables can be cooked, because the water vapor can no longer escape. The pressure is, therefore, increased by the presence of this vapor, and the boiling point of the water is raised.

9. *How is the boiling point of water affected by taking the water to the bottom of a deep mine?*

The boiling point is raised.

10. *In order to free syrup from water without discoloring the sugar, it is necessary to make the water boil at a temperature of about* 150° *F. How can this be done?*

By putting the syrup in a closed pan attached to an air pump. The air pump reduces the pressure in the pan. A pan thus arranged is called a "vacuum pan."

11. *In order to extract gelatine from bones, it is necessary that the water in which the bones are boiled should have a temperature considerably above* 100° *C. How can this be accomplished?*

By putting the bones into a closed vessel. The water in the vessel can then be easily raised to a temperature of more than 100° C.

12. *Why may a vessel made of pewter or other easily fusible metal be safely placed on a hot stove provided it contains some water?*

The evaporation of the water keeps the temperature of the vessel below the melting point of the material of which the vessel is made.

13. *Water may be boiled in a bag made of thin, strong paper without burning the paper. How do you explain this?*

Explained as in Ex. 12. The evaporation keeps the paper below the point of ignition.

14. *If in filling a barometer tube a drop of water enters with the mercury, what effect will this have on the reading of the barometer? Why? Would the effect be greater or less if ether instead of water had been admitted? Why?*

The barometer will read less than it should. The water will instantly evaporate and fill the Torricellian vacuum. The water vapor will exert a pressure on the column, and the mercury will fall. The effect would have been greater if ether had been admitted, because the pressure of ether vapor is greater than that of water vapor.

15. *A cylinder is fitted with a piston, and the confined space within the cylinder is just saturated with water vapor at* 20°. *Describe and explain what will happen in the following cases:*

(1) *If the volume is increased by pulling up the piston.*
(2) *If the volume is diminished by pushing down the piston.*
(3) *If the temperature is raised to* 40°, *the volume remaining unchanged.*
(4) *If the temperature is reduced to* 10°.

(1) The pressure (and density) of the water vapor in the confined space will decrease according to Boyle's Law. The space will no longer be saturated with vapor.

(2) Some of the water vapor will be condensed. The pressure and density of the vapor which does not condense will remain unchanged.

(3) More water will evaporate. The pressure and density of the vapor will increase.

(4) The capacity of the confined space to hold water vapor will be diminished, and a part of the vapor will be condensed. The pressure and density of the vapor will be diminished.

Laboratory Exercises. Page 121.

1. *Find the specific heat of lead shot.*

When taking the temperature of the mixture, you may at first observe somewhat violent fluctuations of the thermometer. After the lapse of a very short time, however, these fluctuations subside. The thermometer is then to be read. Take all the temperatures quickly, but accurately. This rule applies to all readings of the thermometer when used to get the temperatures of substances which are either gaining or losing heat.

2. *Find the latent heat of fusion of ice.*

Use clear ice, broken into small pieces. A good way to break up the ice is to wrap up some large pieces in a cloth, and then beat the cloth with a wooden bar.

3. *Find the latent heat of steam.*

During the experiment, the water in the calorimeter should be stirred with the thermometer. By frequent stirring, the layer of hot water, which floats on the cold water in the calorimeter, is thoroughly mixed with the cold water, and a uniform temperature secured for the whole mass of water.

The weight of the condensed steam is found by subtracting the weight of the calorimeter and the water it originally contained from the weight after the steam was admitted.

Class-room Exercises. Page 123.

1. *Illustrate the fact that heat is a quantity capable of measurement. Show by an example that two bodies may have the same temperature and yet contain very different quantities of heat.*

See § 107. A kilogram of water contains 1,000 times as much heat as 1 gram of water at the same temperature. A pound of water contains about 30 times as much heat as 1 pound of mercury at the same temperature.

2. *Explain the meaning of the statement that the specific heat of water is 30 times as great as that of mercury.*

To raise the temperature of a mass of water 1° requires 30 times as much heat as to raise the temperature of an equal mass of mercury 1°.

3. *Why is the small specific heat of mercury one of its merits as a substance to use for measuring temperatures?*

Because the mercury requires only a small amount of heat to raise its temperature 1°, so that it very quickly responds to a change in temperature.

4. *What is meant by saying that the latent heat of water is 80 units? Mention some of the consequences that would follow if the latent heat of water were only 2 or 3 units.*

The meaning is that it takes 80 calories to change 1 gram of ice at 0° C. into water without change of temperature. See § 110.

5. *Why do tubs of cold water protect a cellar from frost?*

Vegetables freeze at a temperature a little below 0° C. If the temperature of the cellar falls to 0° C., the water begins to freeze. In the process of freezing, heat is given off. Thus the temperature of the cellar is kept from falling below 0° C., and the vegetables do not freeze.

6. *Why do persons often catch cold if they get their clothes damp?*

The heat of the body is taken to raise the temperature of the water-soaked garments and to evaporate the water. See § 113.

7. *On the island of Madeira in the Atlantic Ocean the mean temperature in winter is about 60° F. and in summer about 70° F. How do you account for this small variation in temperature?*

The small annual variation in temperature is chiefly due to the influence of the ocean which surrounds Madeira. The change in temperature of the ocean is small on account of the great capacity of water for heat. The temperature of the land, and especially that of the air above the land, is largely determined by that of the ocean which surrounds the land.

8. *A metal vessel with a thin bottom is placed on a wet board. The vessel is half filled with water, and a quantity of ammonic nitrate is stirred around in the water. Very soon the vessel freezes fast to the board. How is this to be explained?*

When ammonic nitrate is dissolved in water, heat is absorbed. Some

of this heat is taken from the water on the board, and this water, on losing a sufficient amount of heat, changes its state and becomes ice.

9. *Why does ether on the skin cause a sensation of cold?*

When ether evaporates, it quickly absorbs heat. Heat is thus taken from the hand, and the sensation of cold is produced.

10. *Explain the cooling effect produced by watering a dusty road.*

The water in evaporating absorbs a large amount of heat, which lowers the temperature.

11. *Describe some experiment by which a very low temperature can be produced, and explain the production of the cold.*

See § 111.

12. *Men have entered ovens heated to 250° F., where they have remained long enough to cook a steak by blowing hot air on it through a bellows. The men suffered little from the heat beyond a very profuse perspiration. Why were they not more seriously affected by the heat?*

The heat did not seriously affect the men, because it was employed in evaporating the profuse perspiration.

13. *A vessel contains 20 kg. of water at 0°. How many calories of heat must be imparted to the water before it begins to boil?*

$$20000 \times 100 = 2{,}000{,}000 \text{ calories.}$$

14. *How many units of heat are required to raise the temperature of 400 grams of iron from 10° to 200°? Sp. heat of iron = 0.11.*

$$400\,(200 - 10) \times 0.11 = 8{,}360 \text{ calories.}$$

15. *How many heat units are set free when a tank containing 8 tons of water cools from 30° C. to the freezing point?*

One pound = 453.6 grams.

$$8 \times 2000 \times 453.6 \times 30 = 217{,}728{,}000 \text{ calories.}$$

16. *If 7 lb. of water at 25° are mixed with 3 lb. at 65°, what will be the temperature of the mixture?*

Let t denote the temperature of the mixture; then we have, since the amount of heat given up by the 3 lb. is received by the 7 lb.,

$$\begin{aligned} 7\,(t - 25) &= 3\,(65 - t) \\ 7\,t - 175 &= 195 - 3\,t \\ 10\,t &= 370 \\ \therefore t &= 37°. \end{aligned}$$

17. *What is the temperature of a mixture made of 6 kg. of mercury at 20°, and 4 kg. of mercury at 50°?*

Let t denote the required temperature.

Then
$$\begin{aligned} 6\,(t - 20) &= 4\,(50 - t) \\ 3\,t - 60 &= 100 - 2\,t \\ 5\,t &= 160 \\ \therefore t &= 32°. \end{aligned}$$

18. *What temperature will result if we put* 300 *grams of copper at* 100° *into* 200 *grams of water at* 10°? *Sp. heat of copper* = 0.095.

Let t denote the required temperature.

Then, since the amount of heat lost by the copper is gained by the water, we have

$$300\,(100 - t)\,0.095 = 200\,(t - 10)$$
$$28.5 - 0.285\,t = 2\,t - 20$$
$$2.285\,t = 48.5$$
$$t = 21^\circ.22.$$

19. *How many pounds of ice water must be poured into* 40 *lb. of water at* 80° *to reduce the temperature to* 5°?

Let x denote the number of pounds of ice water required.

Then

$$5\,x = 40\,(80 - 5)$$
$$5\,x = 3000$$
$$\therefore x = 600 \text{ lb.}$$

20. *Two pounds of iron at* 100° *are placed in* 3 *lb. of water at* 20°, *and the resulting temperature is* 25°. *Find the specific heat of iron.*

Let x denote the specific heat of iron.

Then

$$2\,(100 - 25)\,x = 3\,(25 - 20)$$
$$150\,x = 15$$
$$10\,x = 1$$
$$\therefore x = 0.1.$$

21. *Find the specific heat of a substance, if* 250 *grams of it at* 78°, *when immersed in* 500 *grams of water at* 12°, *give a temperature of* 18°.

Let x denote the required specific heat.

Then

$$250\,(78 - 18)\,x = 500\,(18 - 12)$$
$$60\,x = 12$$
$$\therefore x = 0.2.$$

22. *If* 60 *grams of iron nails at* 100° *are put into* 136 *grams of water at* 16°, *and the final temperature is* 20°, *find the specific heat of the nails.*

Let x denote the specific heat of the nails.

Then

$$60\,(100 - 20)\,x = 136\,(20 - 16)$$
$$15 \times 80 \times x = 34 \times 4$$
$$150\,x = 17$$
$$\therefore x = 0.11.$$

23. *Five hundred grams of lead shot at* 99° *are poured into a calorimeter which weighs* 78 *grams, and contains* 300 *grams of water at* 14°. *The temperature of the water rises to* 18.5°. *The specific heat of the calorimeter is* 0.1. *Find the specific heat of lead.*

Let x denote the specific heat of lead.

Then $$500\,(99 - 18.5)\,x = 300\,(18.5 - 14) + 78\,(18.5 - 14)\,0.1$$
$$500 \times 80.5 \times x = 307.8 \times 4.5$$
$$50 \times 80.5 \times x = 153.9 \times 0.9$$
$$4025\,x = 138.51$$
$$\therefore x = 0.034.$$

24. *To find the temperature of a furnace, a copper ball weighing* 1 *lb. is put into the furnace, and after some time removed, and dropped into a bucket containing* 20 *lb. of water at* 15° *C. The temperature of the water rises to* 20°. *Taking the specific heat of copper as* 0.1, *find the temperature of the furnace.*

Let t denote the required temperature.

Then $$(t - 20)\,0.1 = 20\,(20 - 15)$$
$$0.1\,t - 2 = 100$$
$$0.1\,t = 102$$
$$\therefore t = 1020°.$$

25. *How much ice at* 0° *will be melted by* 1 *kg. of boiling water?*

Let x denote the number of kilograms of ice melted.

Then $$80\,x = 100$$
$$\therefore x = 1.25 \text{ kg.}$$

26. *Equal weights of hot water and melting ice are mixed. The result is water at* 0°. *What is the temperature of the hot water?*

Let t denote the temperature of the hot water, and let a denote the weight of ice.

Then $$80\,a = a\,t$$
$$\therefore t = 80°.$$

27. *Equal weights of boiling water and melting ice are mixed. The result is water at* 10°. *Hence, find the latent heat of water.*

Let x denote the latent heat of water, and let a denote the weight of the ice.

Then $$ax + 10\,a = a\,(100 - 10)$$
$$x + 10 = 90$$
$$\therefore x = 80 \text{ units.}$$

28. *How many pounds of ice must be mixed with* 6 *lb. of water at* 95° *in order to obtain water at* 10°?

Let x denote the number of pounds of ice required.

Then $$80\,x + 10\,x = 6\,(95 - 10)$$
$$90\,x = 510$$
$$\therefore x = 5.66 \text{ lb.}$$

29. *What will be the resulting temperature* (1) *if* 2 *lb. of ice at* 0° *are mixed with* 10 *lb. of water at* 80°? (2) *if* 10 *lb. of ice at* 0° *are mixed with* 2 *lb. of water at* 80°?

(1) Let t denote the resulting temperature.

$$2 \times 80 + 2t = 10(80 - t)$$
$$80 + t = 400 - 5t$$
$$6t = 320$$
$$\therefore t = 53^\circ.33.$$

(2) Two pounds of ice will be melted, and the resulting temperature will be 0°.

30. *How many inches of rain at 50° F. must fall in order to melt 1 inch of ice at 0°?*

The rain falls in temperature from 50° F. to 32° F., or 18 F. degrees, which are equivalent to 10 C. degrees. Take as the heat unit the quantity of heat given out by 1 cubic inch of rain in falling 1° C. Then a cubic inch of rain in falling from 50° F. to 32° F. will give out 10 units. Since sp. gr. of ice $= 0.92$, it will require 0.92×80 or 73.6 units of heat to melt a cubic inch of ice. Therefore

$73.6 \div 10$ or 7.36 inches of rain must fall.

31. *A calorimeter (sp. heat 0.1) weighing 78 grams contains 200 grams of water at 52°. When 116 grams of ice are put into the water, the temperature falls to 5°. From these data find the latent heat of water.*

Let x denote the latent heat of water.

Then
$$116x + 116 \times 5 = 200(52 - 5) + 78(52 - 5)0.1$$
$$58x + 290 = 4700 + 183.3$$
$$58x = 4593.3$$
$$\therefore x = 79.2.$$

32. *A pound of coal in burning sets free 8,000 units of heat. How much coal is needed to transform 1 ton of water at 0° into steam at 100°?*

Let x denote the number of pounds of coal required.

Then
$$8000x = 2000(100 + 536)$$
$$4x = 636$$
$$\therefore x = 159 \text{ lb.}$$

33. *How much heat is given off when 30 lb. of steam at 100° are cooled down to water at 50°?*

$$30(536 + 50) = 17{,}580 \text{ units.}$$

The unit of heat in this case is the amount of heat required to raise 1 lb. of water 1° C. Number of calories $= 17{,}580 \times 453.6 = 7{,}974{,}288$.

34. *A pound of steam at 100° is blown into 10 lb. of water at 20°. Find the resulting temperature.*

Let t denote the resulting temperature.

Then $$536 + (100 - t) = 10(t - 20)$$
$$636 - t = 10t - 200$$
$$11t = 836$$
$$\therefore t = 76°.$$

35. *A calorimeter (sp. heat 0.1) weighing 78 grams contains 300 grams of water at 4°. In this water 23 grams of steam at 100° are condensed. The temperature of the water rises to 48°. Find the latent heat of steam.*

Let x denote the latent heat of steam.

Then $$23x + 23(100 - 48) = 300(48 - 4) + 78(48 - 4)\,0.1$$
$$23x + 1196 = 13200 + 343.2$$
$$23x = 12347.2$$
$$\therefore x = 536.8.$$

Class-room Exercises. Page 135.

1. *How is the air of a room heated by a closed stove?*

By convection.

2. *On the Mer de Glace at Chamouni, Switzerland, Count Rumford found holes in the ice about 4 ft. deep and 7 in. wide filled with water which deepened in summer from day to day. If water is a non-conductor of heat, how can this melting of the ice at the bottom occur?*

Water at 4° C. kept sinking to the bottom of the holes, and heat from this water melted the ice at the bottom.

3. *Viscous liquids, such as molasses, oil, and thick soups, cool more slowly than water. Can you think of any explanation?*

Viscosity tends to prevent convection currents.

4. *If you enter a cold room and touch first the fire irons, then the chimney flue, then the chairs, and then the carpet, how will they feel as regards temperature? How, also, in case the room is hot instead of cold? Explain.*

In the first case, each object in the list will appear colder than the succeeding one. In the second case, each object will appear warmer than the succeeding one; because in the list of objects the best conductors are placed first, the poorest last.

5. *Sawdust is the same material as wood, but a poorer conductor of heat. Explain why.*

Air fills the spaces between the particles of sawdust, and air is a very poor conductor of heat.

6. *Why are kitchen utensils often provided with wooden handles?*

To protect the hands from the heat, since wood is a poor conductor.

7. *Why does the lock of a door feel colder than the wood of which the door is made?*

Because the lock is a better conductor of heat, and therefore takes heat from the skin more rapidly than the wood.

8. *Explain the use of double windows.*

A layer of air, a poor conductor of heat, is imprisoned between the double windows; consequently the heat of the room does not escape so quickly.

9. *Why does woolen clothing keep us warm in winter?*

Wool is a poor conductor of heat. Woolen clothing keeps the heat of the body from escaping.

10. *Why do farmers like to have the ground covered with snow during the winter?*

Because snow is a poor conductor of heat, and therefore a thick covering of snow keeps the ground from freezing to any great depth.

11. *Tyndall, in one of his experiments, held a red-hot iron ball in his hand, first taking care to lay in his hand a sheet of asbestos paper. Why was he not burned?*

Because asbestos paper is a very poor conductor of heat.

12. *In winter, ice covered with ashes melts sooner than ice freely exposed. In summer just the reverse happens. Explain these facts.*

In winter the melting of ice is mostly due to the action of the sun's rays. Suppose that two pieces of ice are lying on the ground, one piece bare, the other covered with ashes. If the sun is shining, the ashes absorb much heat, and the consequence is that the ice underneath the ashes melts faster than the ice freely exposed. It is true that the sun is shining directly upon the bare ice; but some of the heat is reflected away from the ice, and some is transmitted through the ice without melting it. Moreover, the cold air, which is in direct contact with the bare ice, tends to prevent it from melting. Another fact of importance is that after the sun has set, the bare ice, being in direct contact with the cold air, ceases to melt; while the ice covered with the warm ashes continues for some time to melt.

In summer we try to keep ice from melting. We keep it in a room where the sun cannot shine on it. The temperature of the room, of course, is much above 0°. Under these conditions ice covered with ashes will not melt so fast as bare ice, because ashes is one of the best non-conductors of heat.

13. *Why does water boil sooner in a kettle covered with soot than in one brightly polished?*

Because soot is an excellent absorber of heat.

14. *Why does cold, damp air chill a man more than dry air which is still colder?*

It takes more heat to raise the temperature of damp air 1° C. than to raise the temperature of the same weight of dry air 1° C. Hence, more heat passes in the same time from the body to cold, damp air than to dry air; and this is true even if the temperature of the dry air is considerably below that of the damp air.

15. *Why does the glass covering of a conservatory make the air inside warmer than the air outside?*

See § 124.

16. *Arctic navigators have observed that in summer snow surrounding a black object melts, although the temperature of the air is below 0° C. How is this fact explained?*

The black object absorbs the sun's heat very freely, and then radiates heat, which gradually melts the snow lying very near the object.

17. *It is a well-known fact that snow melts around the trunk of a tree sooner than in an open field. How do you explain this fact?*

The trunk of the tree absorbs heat during the day and radiates it during the night. The absorption and subsequent radiation of this heat is the reason why snow near the trunk melts sooner than snow in an open field.

Review Exercises on Chapter III. Page 141.

1. *If a copper vessel tightly corked were placed over a flame, and if you very soon saw the cork driven violently from the vessel, what would you infer was in the vessel?*

Water which was converted into steam by the heat.

2. *How would you proceed in order to ascertain which liquid, water or alcohol, expands at the faster rate when heated?*

Fill two bulb tubes equal in size, one with water and the other with alcohol, and mark the position of the top of the liquid column in each tube, when the two liquids have the same temperature. Plunge both bulbs into a vessel of hot water. By observing the increase in the heights of the liquid columns, we learn that alcohol expands faster than water.

3. *Give an example in which the expansion of a solid is put to a useful purpose.*

The thermometer.

4. *Give an example in which the expansion of a solid has to be allowed for.*

In laying an iron track spaces have to be left between the rails.

5. *Explain what is meant by saying that the coefficient of linear expansion of iron is* 0.000012.

The meaning is that if a bar of iron is heated from 0° C. to 1° C., the increase in its length is 0.000012 of its length at 0° C.

6. *Prove that the coefficient of cubical expansion of a body is equal to three times its coefficient of linear expansion* (§ 96).

See § 96.

7. *What is meant by saying that the maximum density of water is at 4° C.? How was this fact proved by Hope?*

See § 95.

8. *A bridge is built in winter, and the girders are laid end to end so that they touch one another. What will happen to them in summer?*

In summer the girders will expand and be thrown out of line.

9. *What effect has the heat of summer on the rate of a clock, and why? How can you make a pendulum that will keep correct time both in summer and in winter?*

The heat of summer makes the rate of a clock slower, because the pendulum increases in length. For one kind of compensation pendulum see page 104, Exercise 4.

10. *Why is mercury commonly employed to measure temperature. Why is air better than mercury for the accurate measurement of temperature?*

Mercury is a good conductor of heat, and very quickly responds to a change of temperature. It is also nearly uniform in its rate of expansion. For the merits of air as a thermometric substance see § 97.

11. *State Charles's Law* (1) *when a gas is heated under constant pressure, and* (2) *when a gas is heated under constant volume.*

See § 97.

12. *Explain what is meant by the absolute zero of temperature. What is it on the Centigrade scale? What is it on the Fahrenheit scale?*

See § 98. Absolute zero Centigrade $= -273°$. Absolute zero Fahrenheit $= \frac{9}{5}(-273) + 32 = -459.4°$.

13. *State the laws of Fusion.*

See § 99.

14. *Why do water pipes sometimes burst in cold weather?*

Because, when the water freezes, it expands with force sufficient to burst the pipe.

15. *What is the meaning of regelation? Give an example.*

See § 101.

16. *On what circumstances does the rate of evaporation of a liquid depend?*

See § 103.

17. *State the laws of Ebullition.*

See § 104.

18. *Why cannot vegetables be cooked at high altitudes in open vessels? How can they be cooked in such places?*

The water boils at too low a temperature. The vegetables can be cooked in closed vessels.

19. *How would you obtain pure water from sea water?*

By distillation. See § 106.

20. *Define a unit of heat, supposing a Fahrenheit thermometer is used, and that the unit of mass of water is one pound.*

The quantity of heat required to raise the temperature of 1 lb. of water 1° F.

21. *What is meant by saying that the specific heat of iron is* 0.11?

The meaning is that it requires only 0.11 as much heat to raise the temperature of a piece of iron 1° C. as to raise the temperature of the same weight of water 1° C.

22. *How does the great specific heat of water benefit human beings?*

See § 108.

23. *What is meant by saying that the latent heat of water is* 80 *units?*

The meaning is that it requires 80 times as much heat to change ice at 0° into water at 0° as to raise the temperature of the same weight of water 1°.

24. *Explain the action of a freezing mixture.*

See § 111.

25. *Give an example of the latent heat of evaporation.*

See § 112.

26. *What is meant by the statement that the latent heat of steam is* 537 *units?*

The meaning is that it requires 537 times as much heat to change water at 100° into steam at 100° as to raise the temperature of an equal weight of water 1°.

27. *Why does your hand feel cool if you dip it in warm water and then expose it to equally warm air?*

Because the water evaporates and takes heat from the hand.

28. *How can water be made to boil at a temperature less than* 100° *C.?*

By heating water under less than atmospheric pressure.

29. *State briefly in what ways convection, conduction, and radiation differ from one another.*

In convection there is a flow of heated matter (liquid or gas) to places of lower temperature. In conduction the matter composing the body

remains sensibly at rest, and there is an apparent flow of heat to places of lower temperature. In radiation heat passes in straight lines from a hot body through the matter which surrounds it without necessarily raising the temperature of the matter through which it passes.

The transference of heat by convection or by conduction is comparatively a slow process. In the case of radiation, the heat moves through space with a velocity equal to that of light (186,000 miles per second).

30. *Give an example of convection currents usefully applied.*

Heating a building by hot-water pipes.

31. *How would you compare the conducting powers of two metals?*

See § 119.

32. *How would you show that mercury conducts heat better than water?*

Fill two similar tubes, one with water and the other with mercury. Hold one tube by the lower end in one hand, and the other tube by the lower end in the other hand, and stick their upper ends in the flame of a lamp. The lower end of the tube containing the mercury will very soon become hot, while the lower end of the other tube will remain sensibly at the same temperature.

33. *Two similar bars of copper and lead are coated with wax. One end of each bar is then exposed to the same source of heat (see Fig.* 125). *At first more wax is melted on the lead than on the copper; after some time more wax is melted on the copper than on the lead. Explain this.*

At first more wax is melted on the lead because the specific heat of lead is less than that of copper; but finally the wax melts to a greater distance on the copper. This proves that copper conducts heat better than lead. See § 119.

34. *Which is better for the back of a fireplace, brick or polished metal?*

Brick, because it radiates heat much better than polished metal.

35. *How do radiation, absorption, and reflection differ from one another?*

See §§ 121 and 123.

36. *Which will hold the heat longer, a polished kettle, or a kettle covered with rust?*

A polished kettle, because it does not radiate heat so well as a rusty kettle.

37. *The glass of a greenhouse has been called "a trap to catch the sunbeams." Why?*

The luminous heat of the sun can pass through the glass roof of the

greenhouse and warm the objects within; but the dark heat radiated by these objects cannot pass through the glass and escape to the air outside. See § 124.

38. *What are the conditions that favor the formation of dew?*

See § 124.

39. *Define the dew point, and explain how you would find it by experiment?*

See § 128.

40. *Why does dew form copiously on grass, but scarcely at all on gravel?*

Because grass radiates heat much better than gravel.

41. *Why does dew form more copiously on a clear night than on a cloudy night?*

On a clear night the heat radiated from the earth escapes into space, and there is no return radiation. On a cloudy night the clouds radiate back to the earth more or less heat. Therefore on a cloudy night the temperature of the ground does not fall so rapidly as on a clear night. Therefore less moisture is condensed on the ground in the form of dew.

42. *A knife with a black handle is left in the open air over night. Which part of the knife, the blade or the handle, will be covered with dew in the morning? Explain the reason.*

The handle, because it radiates heat better than the bright, polished blade.

43. *Why does the mercury fall if a thermometer is carried up into the air?*

Because, as we ascend in the air, the temperature of the air decreases. See § 126.

44. *Explain the production of land breezes and of sea breezes.*

During the night the land loses heat by radiation, and becomes cooler, but the temperature of the water remains practically unchanged. The water does not radiate heat so well as the land, and water also has a greater capacity for heat than the land. Consequently the warmer air over the water rises, and makes room for the colder air over the land, which flows seaward. This current of air is the land breeze. During the day the land, by absorbing heat, rises rapidly in temperature, and communicates its heat to the air which is directly over it. This heated air rises, and then the cooler air over the ocean flows towards the land to take its place. This current of air, which is most noticeable in the afternoon, forms what is known as the sea breeze.

CHAPTER IV.

MATTER.

Class-room Exercises. Page 152.

1. *Why does stirring hasten the solution of a solid in water?*
By stirring, the solid is brought in contact with fresh portions of water; these act more quickly in effecting solution than the portions of water which already contain some of the solid in solution.

2. *What would you do in order to make a pint of water dissolve as much of a solid as possible? And what in order to make the water dissolve the solid as quickly as possible?*
Heat the water. Pulverize the solid, put the powder into the water, and stir the water constantly.

3. *Why does a solid dissolve more readily in the form of powder than in the form of a solid lump?*
Because more surface is exposed to the action of the liquid.

4. *Does the fact that we can thrust our finger into water, or drive a nail into wood, conflict with the statement that matter is impenetrable?*
No. By so doing we merely push the molecules aside.

5. *What is the reason that water will not run out of a tightly fitting funnel into an empty bottle?*
The pressure of the air in the bottle prevents the water from running in.

6. *Why is it necessary, in making good barometers, to boil the mercury contained in the tube?*
In order to expel the air, which is absorbed by the mercury.

7. *Doors that open freely in winter are sometimes found to stick in summer. What is the explanation?*
The doors in summer absorb moisture from the air and increase slightly in size, or "swell," to use the common expression.

8. *When a vessel has sunk to the bottom of the sea, and been there for some time, the wooden parts when set free will not rise to the surface. What is the explanation?*
The air between the particles of the wood has been replaced by water.

The wood thus "water-logged" is heavier, bulk for bulk, than the water, consequently the wood does not rise.

9. *How do you explain the crackling of wood when it burns?*

The air in the pores of the wood expands when heated, and the bursting of the walls of the cells containing the air produces the crackling.

10. *How do you explain the coloring of a meerschaum pipe?*

The tobacco juice slowly soaks into the pores of the bowl and stem of the pipe, and gives them a brownish tint.

11. *What in general is the effect of heat on the capacity of water (1) to dissolve a solid, (2) to dissolve a gas?*

Heat in general increases the capacity of water to dissolve a solid, but diminishes its capacity for dissolving a gas.

12. *How would you separate common salt and sand if they were mixed together?*

Pour water on the mixture to dissolve the salt. Then pour the salt solution from the vessel, leaving the sand behind. By the process of filtering the salt and the sand can be separated more completely. On evaporating the solution, the salt will be obtained in a solid state.

13. *Persons who camp out in the woods sometimes find their supply of salt mixed with dirt. How can they free it from the dirt?*

By the method described in the preceding answer.

14. *Mention facts or phenomena which show that water is capable of holding gases in solution.*

See § 137.

15. *Which will diffuse the more rapidly through a membrane by osmotic action, common salt or gelatine?*

Common salt.

Laboratory Exercises. Page 160.

1. *Find the breaking strength of brass wire, using size No. 30, diameter 0.255 cm., Brown & Sharpe's gauge, and a 24-lb. spring balance.*

Make several trials of the breaking strength, using a fresh piece of wire each time. The average of the results obtained should be taken as the breaking strength of the wire used. Pieces of wire about 1 meter long are convenient to use in this experiment. One end of the wire should be wound round some fixed cylindrical object, such as a gas pipe, and fastened by twirling the end of the wire round a peg or tack. The other end should be fastened to the eye which holds the hook of the balance. The wire should be then wound round some cylindrical object which has been slipped over the hook, and which cannot turn on

the hook. Pull steadily on the balance, which should be held in a horizontal position. Watch the pointer carefully, and note its reading when the wire breaks. The wire should be free from kinks. If the wire breaks at the cylinders round which it is wound at each end, no record should be made of the reading of the balance. The windings of the wire round the cylinders should not overlap, lest the wire be cut by the pressure.

2. *Place two tumblers, one higher than the other. Connect them by a cotton wick. Pour some water into the upper glass. What will happen? Write out an explanation of this effect.*

The water will pass from the higher tumbler to the lower through the wick. The water rises by capillary action among the fibers of the wick when it dips into the upper tumbler. The water, by the same action, passes round the bend in the wick, and descends to the other tumbler into which it drips. We now have a siphon, and the water will flow till either the surface of the water in the upper tumbler falls below the end of the wick in that tumbler, or till the level of the water in the lower tumbler rises to the level of that in the upper tumbler.

3. *Experiments with soap bubbles and films.*

Modify the experiment of Fig. 141, § 145, by knotting the thread so as to make a small ring. Drop this ring upon the film, and by a hot iron wire break the part of the film inclosed by the thread. The thread will immediately take the form of a circumference. For various experiments with soap bubbles and films, see Boys's "Soap Bubbles and the Forces Which Mould Them."

4. *Experiments on capillary elevation and depression in glass tubes of different diameters.*

See that the tubes are not stopped up. By holding a sheet of white paper behind the tubes, the water in them can more easily be seen.

Class-room Exercises. Page 160.

1. *How is the tenacity or breaking strength of a substance measured? Name a substance having very great tenacity.*

By the tensile stress that will break a bar of the material the area of the cross-section of which is 1 sq. in. Steel has very great tenacity.

2. *Explain the meaning of the terms: ductile, malleable, brittle.*

See § 143.

3. *Describe annealing and tempering. What purpose do they serve?*

See § 143.

4. *Explain the rise of oil in the wick of a lamp.*

The wick has a great number of capillary tubes running through it from one end to the other. It is through these tubes that the liquid rises by capillary action.

5. *Explain why two pieces of wood floating near each other in water are attracted to each other.*

The water between the two pieces of wood is raised slightly at the edges, where it is in contact with the wood. The superficial liquid film between the pieces of wood tends to contract, and the result of this tendency is at first a slight movement of one piece of wood towards the other. But as this tendency continues to exist, and becomes stronger as the pieces of wood approach each other, the two pieces of wood continue to approach till they meet.

6. *Explain the action of blotting paper in removing ink spots.*

Blotting paper is of an open texture, and contains many capillary tubes, into which the ink rises when the paper is applied to an ink spot.

7. *Give examples of capillary phenomena.*

The absorption of ink by blotting paper, the rise of coffee in a lump of sugar, the absorption of water by a sponge and by cloths, the depression of mercury in a tube plunged into a cup of mercury, the rise of water at the sides of a floating block of wood if the block is not oiled, and the depression of the water if the block is oiled.

8. *Two capillary tubes dip into water. The diameter of one is half that of the other. The water rises in the larger tube* 1 cm. *How far will it rise in the smaller tube?*

The water in the smaller tube will rise 2 cm. See law 3, § 147.

9. *Which of the following liquids are viscous and which mobile: castor oil, water, glycerine, ether, alcohol, molasses, tar?*

Castor oil, glycerine, molasses, and tar are viscous liquids. Water, ether, and alcohol are mobile liquids.

Review Exercises on Chapter IV. Page 172.

1. *Sum up the molecular theory of matter in four propositions.*

See § 133.

2. *What is meant by a saturated solution of a substance?*

A solution which cannot dissolve any more of the substance.

3. *What is the usual effect of temperature on solubility?*

A rise of temperature, in general, increases the solubility.

4. *What conditions favor the formation of large, perfect crystals?*

(1) The liquid must be kept at rest. (2) The liquid must be cooled, or evaporated very slowly.

5. *Illustrate the uses of crystallization.*

See bottom of page 146.

6. *Give an instance of a solid having a crystalline structure, and name the kind of structure.*

See bottom of page 147.

7. *Give an example of the power of a solid to absorb a gas.*

See § 137, 1.

8. *Give an example of the power of a liquid to absorb a gas.*

See § 137, 2.

9. *Give an example of liquid diffusion, and show how it tends to establish the molecular theory of matter.*

See § 138.

10. *Define with illustrations the meaning of the terms osmosis, crystalloid, colloid, dialysis.*

See § 139.

11. *Give examples of the power of gases to mix with one another.*

See § 140.

12. *What is the effect of heat on cohesion? Give an example.*

See § 142.

13. *Explain with examples the meaning of the terms elasticity, plasticity, brittleness, toughness, malleability, ductility, hardness.*

See § 143. Steel properly tempered is very elastic; moist clay or dough is very plastic; glass is very brittle; wrought iron is very tough; gold is very malleable; platinum is very ductile; the diamond is the hardest substance known.

14. *Give examples of cohesion in liquids.*

See § 144.

15. *How can a liquid be shown to have a surface tension.*

See § 145.

16. *Give examples of adhesion* (1) *between two solids,* (2) *between a solid and a gas,* (3) *between a solid and a liquid.*

See § 146.

17. *What are capillary phenomena and their chief laws?*

See § 147.

18. *Give examples of viscous and of mobile liquids. How do we distinguish between solids and liquids?*

Pitch, tar, and molasses are examples of viscous liquids. Ether, chloroform, alcohol, and water are examples of mobile liquids.

A solid is a body that will preserve its form unless a sufficiently great force is brought to bear upon it. When the force is sufficiently great,

the form of the body changes at once. A liquid, on the other hand, is a substance which the slightest force will change in form, provided the force acts long enough.

19. *Illustrate by examples the difference between a mechanical mixture and a chemical compound.*

If iron filings and sulphur are mixed together, we have a mechanical mixture. The iron and the sulphur can be separated from each other by mechanical means. We can either withdraw the iron filings from the sulphur by a magnet, or we can withdraw the sulphur by dissolving it in bisulphide of carbon.

If we apply a hot flame to the mechanical mixture of iron filings and sulphur, a glow spreads through the whole mass. A chemical change takes place, and a chemical compound is formed. On examining the residue, we should find, besides a possible excess of either iron or sulphur, a substance resembling neither iron nor sulphur, and one in which we cannot separate the iron and the sulphur by mechanical means. The iron and the sulphur have united in such a way that a new substance called iron sulphide is formed.

20. *Explain how water can be decomposed into oxygen and hydrogen.*

See § 151.

21. *Distinguish between an element and a compound, and give examples.*

A substance which cannot be decomposed into other substances is called an *element.* A substance which can be decomposed into other substances is called a *compound.*

The metals, oxygen, hydrogen, sulphur, and carbon are examples of elements. Water, common salt, and sulphuric acid are examples of compounds.

22. *Give an example of chemical exchange.*

See § 153.

23. *State the general laws of chemical combination.*

See § 154.

24. *Give a brief account of the atomic theory.*

See § 155.

25. *How did Lavoisier show that the air we breathe is a mixture of oxygen and nitrogen gases?*

See § 157.

26. *What is the general effect of chemical combination and chemical separation as regards heat? Give an example of oxidation, and also one of combustion.*

In general, heat is set free when chemical combination takes place, but absorbed when chemical separation takes place.

The rusting of iron is an example of oxidation. The burning of a candle is an example of combustion.

27. *Describe how animal heat is maintained.*

Animal heat is maintained by the chemical combinations which are all the time going on in the cells of the body of a living animal.

28. *Describe how the reduction of carbon dioxide is effected by the plant world.*

See § 160.

CHAPTER V.

MOTION.

Class-room Exercises. Page 178.

1. *How far will an eagle fly in 1 hour at the rate of 100 ft. per sec.?*
Distance $= 60 \times 60 \times 100 = 360{,}000$ feet $= 68.18$ miles.

2. *A velocity of 60 miles an hour is how many feet per second?*

$$\frac{60 \times 5280}{60 \times 60} = 88 \text{ ft. per second.}$$

3. *A train leaves Boston at 2 P.M., and reaches Chicago the next day at 5 P.M. The distance is 1025 miles. What is its average rate?*

$$\text{Average rate} = \frac{1025}{27} = 38 \text{ miles per hour, nearly.}$$

4. *A man looking out of the window of a car observes that the time between passing one milestone and the next is 80 seconds. What is the velocity of the train in miles per hour?*

As the train goes one mile in 80 seconds, it will go in one hour, or $60 \times 60 = 3600$ seconds, as many miles as 80 is contained times in 3600, or 45 miles.

5. *A cannon is fired on the water one mile from a cliff, and the sound of the echo is heard after $9\frac{1}{2}$ seconds. Find the velocity of sound.*

The sound travels to the cliff and back again, a distance of 2 miles, in $9\frac{1}{2}$ seconds. Hence we have

$$\text{Velocity of sound} = \frac{2 \times 5280}{9\frac{1}{2}} = \frac{4 \times 5280}{19} = 1111.6 \text{ ft. per sec.}$$

6. *The Danish astronomer, Roemer, found that it took light 16 min., 26 sec. to cross the earth's orbit, a distance of 186 millions of miles. Find the velocity of light in miles per second.*

$$\frac{186000000}{16 \times 60 + 26} = \frac{186000000}{986} = 189{,}000 \text{ miles per sec. approximately.}$$

7. *A steamer can go down a river at the rate of 16 miles an hour, and up the river at the rate of 4 miles an hour. What is the rate of the current, and the rate of the steamer in still water?*

If x denotes the rate of the current, $16 - x$ will denote the rate of the steamer in still water.

Hence rate at which steamer goes up river $= 16 - 2x$.

The rate at which steamer goes up river $= 4$.

Hence $16 - 2x = 4$; $\therefore$ $x = 6$, and $16 - x = 10$.

Rate of current 6 miles an hour. Rate of steamer 10 miles an hour.

8. *A sledge party is traveling on the ice towards the North Pole at the rate of* 16 *miles a day. The ice is drifting southwards at the rate of* 22 *yards a minute. In what direction and at what rate is the party really moving?*

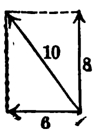
$$\text{Rate of party north} = \frac{16 \times 1760}{24 \times 60} = 19\tfrac{5}{9} \text{ yds. per minute.}$$

Rate of party south $= 22$ yds. per minute.

Hence the party is drifting $22 - 19\frac{5}{9} = 2\frac{4}{9}$ yds. per minute towards the south.

9. *How grea tis the change of velocity from* 28 *east to* 20 *west?*

$$28 + 20 = 48.$$



10. *Find the resultant of the following velocities:* 12 *north,* 2 *east,* 4 *south,* 8 *west, and show its direction by a figure.*

$$12 - 4 = 8 \text{ north}$$

$$8 - 2 = 6 \text{ west.}$$

$$\text{Resultant} = \sqrt{8^2 + 6^2} = 10.$$

11. *A steamer is driven east by the steam at the rate of* 15 *miles an hour, and north by a current at the rate of* 8 *miles an hour. What is its actual velocity relative to the earth?*

$$\text{Resultant} = \sqrt{15^2 + 8^2} = 17 \text{ miles an hour.}$$

12. *A ship is making* 12 *miles an hour on a northeast course. How fast is she moving north? How fast is she moving east?*

As the ship is moving due northeast, her rate towards the north is the same as her rate towards the east.

Let x denote her rate east.

Then $2x^2 = 144$, whence $x = \sqrt{72} = 6\sqrt{2} = 8.484$ miles an hour.

The rate north is also 8.484 miles an hour.

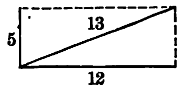
13. *A man rows directly across a river* 2 *miles wide. He rows at the rate of* 6 *miles an hour, and the current flows at the rate of* 3 *miles an hour. How long will he be in crossing, and where will he land?*

$$\text{Time of crossing} = \tfrac{2}{6} = \tfrac{1}{3} \text{ hour.}$$

In $\frac{1}{3}$ hour the current would carry the man 1 mile down stream.

Hence the man will land on the opposite bank of the river 1 mile below the point from which he started.

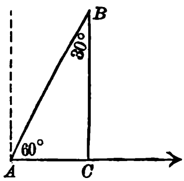
14. *A ship goes* 12 *miles an hour, and a man walks straight across the deck at the rate of* 5 *miles an hour. What is his velocity relative to the water? Illustrate by a figure.*



Resultant $= \sqrt{12^2 + 5^2} = 13$ miles an hour.

15. *A balloon ascends along a line inclined* 60° *to the horizon with a velocity of* 20 *miles an hour. What is its velocity estimated in a horizontal direction?*

Let AB represent the velocity of the balloon. Draw AC horizontal and BC vertical. Then AC represents the horizontal component of the velocity, and BC represents the vertical component. Since the angle $ABC = 30°$, AC is equal to half of AB or 10.



Answer, 10 miles per hour.

16. *If the minute hand of a clock is* 6 *in. long, what is the linear velocity of the end of the hand? What is its angular velocity (that is, the angle it will describe in one second of time)?*

Circumference traced out by the end of the hand $= 2 \times \frac{22}{7} \times 6$ in. This distance is traversed in 1 hour, or $60 \times 60 = 3600$ seconds.

$$\text{Hence, velocity of end of hand} = \frac{2 \times 22 \times 6}{7 \times 3600} = \frac{11}{1050} \text{ in. per sec.}$$

The hand moves through an angle of 360° in an hour, or 3600 seconds. Hence, the angular velocity of the hand $= \frac{360}{3600} = \frac{1}{10}$ of a degree, or 6′.

Class-room Exercises. Page 186.

Values of g : 32 ft. per sec. each second, 9.8 m. per sec. each second.

1. *What velocity will a body acquire in falling for* 10 *seconds, and through what distance will it fall?*

In English units $v = \frac{1}{2}$ gt. $= 32 \times 10 = 320$ ft. per sec.
And $s = \frac{1}{2}$ gt.$^2 = \frac{1}{2} \times 32 \times 10^2 = 1600$ ft.
In metric units $v = 9.8 \times 10 = 98$ m. per sec.
And $s = \frac{1}{2} \times 9.8 \times 10^2 = 490$ m.

2. *How far will a body fall in* 4 *seconds? How far during the fifth second? What is its mean velocity during the* 5 *seconds?*

Let s and v respectively denote the space described and the velocity acquired in 4 seconds; s' and v' the space described and the velocity acquired in 5 seconds.

In English units, $s = \frac{1}{2} \times 32 \times 4^2 = 256$ ft.
$s' = \frac{1}{2} \times 32 \times 5^2 = 400$ ft.
$v = 32 \times 4 = 128$ ft. per sec.
$v' = 32 \times 5 = 160$ ft. per sec.

$\therefore$ space described in the fifth second $= s' - s = 144$ ft.

Mean velocity during the fifth second $= \frac{1}{2}(128 + 160) = 144$ ft. per sec.

In metric units, $s = \frac{1}{2} \times 9.8 \times 4^2 = 78.4$ m.
$s' = \frac{1}{2} \times 9.8 \times 5^2 = 122.5$ m.
$v = 9.8 \times 4 = 39.2$ m. per sec.
$v' = 9.8 \times 5 = 49$ m. per sec.

$\therefore$ Space described in fifth second $= 44.1$ m.

Mean velocity for fifth second $= 44.1$ m. per sec.

3. *How long will it take a body to fall 400 ft., and what velocity will the body acquire? What is its mean velocity?*

$$400 = \frac{1}{2} \times 32 \times t^2; \therefore t = 5 \text{ sec.}$$
$$v = 32 \times 5 = 160 \text{ ft. per sec.}$$
$$\text{Mean velocity} = \frac{0 + 160}{2} = 80 \text{ ft. per sec.}$$

4. *How far must a body fall to acquire a velocity of 96 ft. a second?*

Substituting in the formula $v^2 = 2\,g\,s$, we have

$$96^2 = 2 \times 32 \times s; \therefore s = 144 \text{ ft.}$$

5. *A stone is dropped into a mine, and reaches the bottom in 6 seconds. How deep is the mine?*

$$s = \frac{1}{2} \times 32 \times 6^2 = 576 \text{ ft.}$$

The depth of the mine is 576 ft.

6. *A stone is thrown down a well with a velocity of 50 ft. per second, and reaches the bottom in 2 seconds. How deep is the well?*

$$s = vt + \frac{1}{2}gt^2 = 50 \times 2 + \frac{1}{2} \times 32 \times 2^2 = 164 \text{ ft.}$$

The depth of the well is 164 ft.

7. *Through what height must a body fall to acquire a velocity of 1,000 ft. per second?*

$$1000^2 = 2 \times 32 \times s; \therefore s = 15{,}625 \text{ ft.}$$

The body must fall from a height of 15,625 ft.

8. *A body is dropped from a certain point, and a second later another body is dropped from the same point. How far apart are the two bodies when the first body has been falling for 7 seconds?*

Total distance described by first body in 7 sec. $= \frac{1}{2} \times 32 \times 7^2$.

Total distance described by second body in 6 sec. $= \frac{1}{2} \times 32 \times 6^2$.

Distance apart of the bodies $= 16 \times (7^2 - 6^2) = 16(49 - 36) = 208$ ft.

9. *A stone is thrown vertically upwards with a velocity of 192 ft. per second. How high will it rise? How long will it be in the air?*

Height ascended $= \dfrac{u^2}{2g}$.

Hence height to which body will rise $= \dfrac{192^2}{2 \times 32} = 576$ ft.

Time of rising to highest point $= \dfrac{u}{g} = \dfrac{192}{32}$.

As the body will take as long to fall as to rise, we have for the time that the body will be in the air $\dfrac{2 \times 192}{32} = 12$ sec.

10. *With what velocity must a stone be thrown up in order that it may rise* 1600 *ft.?*

$$1600 = \frac{u^2}{2 \times 32}, \text{ whence } 40 = \frac{u}{8};\ \therefore u = 320 \text{ ft. per sec.}$$

11. *A stone thrown vertically upwards returns to the ground after* 10 *seconds. Find* (1) *velocity of projection,* (2) *height to which it rises,* (3) *height from the ground after* 1, 2, 3, 4, *and* 5 *seconds respectively.*

(1) It will take the stone 5 sec. to rise to its greatest height. Hence

$$5 = \frac{u}{32};\ \therefore u = 160 \text{ ft. per sec.}$$

(2) Height ascended $= \dfrac{160^2}{2 \times 32} = 400$ ft.

(3) By substitution in the formula $s = ut - \frac{1}{2} g t^2$.

We find for the height above the ground

After 1 second	$160 \times 1 - \frac{1}{2} \times 32 \times 1^2 = 144$ ft.
After 2 seconds	$160 \times 2 - \frac{1}{2} \times 32 \times 2^2 = 256$ ft.
After 3 seconds	$160 \times 3 - \frac{1}{2} \times 32 \times 3^2 = 336$ ft.
After 4 seconds	$160 \times 4 - \frac{1}{2} \times 32 \times 4^2 = 384$ ft.
After 5 seconds	$160 \times 5 - \frac{1}{2} \times 32 \times 5^2 = 400$ ft.

12. *A stone dropped from a balloon reaches the ground in* 20 *seconds. How high is the balloon,* (1) *if at rest in the air,* (2) *if ascending with a velocity of* 160 *ft. per second when the stone is dropped?*

(1) $s = \frac{1}{2} \times 32 \times 20^2 = 6{,}400$ ft.

(2) $s = \frac{1}{2} \times 32 \times 20^2 - 160 \times 20 = 3{,}200$ ft.

13. *A bullet is fired vertically upwards, with a velocity of* 800 *ft. per second. How high will it rise? How high will it be after* 30 *seconds, and what will be its velocity at this instant?*

Height ascended $= \dfrac{800^2}{2 \times 32} = 10{,}000$ ft.

Height after 30 sec. $= 800 \times 30 - \frac{1}{2} \times 32 \times 30^2 = 9{,}600$ ft.

Velocity after 30 sec. $= 800 - 32 \times 30 = -160$ ft. per sec.

The minus sign denotes that the body is falling.

14. *A stone is dropped into a mine, and the sound, when it strikes the bottom, is heard after* 10 *seconds. If the velocity of sound is* 1120 *ft. per second, find the depth of the mine.*

Let t be the time taken by the stone in falling, then $10 - t$ will be the time taken by the sound in traveling a distance equal to the depth of the mine. We have

$$s = \tfrac{1}{2} \times 32 \times t^2$$

and
$$s = (10 - t)\, 1120$$
$$\therefore 16\, t^2 = (10 - t)\, 1120$$
$$t^2 = (10 - t)\, 70$$
$$t^2 + 70\, t = 700$$
$$t + 35 = \pm \sqrt{700 + 1225} = \pm 5\sqrt{77} = \pm 43.875$$
$$t = -35 + 43.87 = 8.875$$
$$s = (10 - t) \times 1120 = 1260 \text{ ft.}$$

15. *A stone is dropped down a well* 400 *ft. deep. If the sound of the splash is heard after* $5\frac{4}{11}$ *seconds, find the velocity of sound.*

$$400 = \tfrac{1}{2} \times 32 \times t^2, \text{ whence } t = 5 \text{ sec.}$$

Hence the stone will take 5 sec. in falling, and consequently the sound will take $5\frac{4}{11} - 5 = \frac{4}{11}$ of a second in traveling 400 ft. Therefore, velocity of sound $= 400 \div \frac{4}{11} = 1100$ ft. per second.

16. *A body is thrown vertically upwards with a velocity of* 49 *meters per second. With what velocity will it pass a point* 100 *meters from the ground* (1) *when ascending,* (2) *when descending?*

$$100 = 49 \times t - \tfrac{1}{2} \times 9.8 \times t^2$$
$$4.9\, t^2 - 49\, t + 100 = 0$$
$$t = \frac{49 \pm \sqrt{49^2 - 4 \times 490}}{9.8} = \frac{49 \pm 21}{9.8} = \frac{28}{9.8} \text{ or } \frac{70}{9.8}.$$

When $t = \frac{28}{9.8}$ sec. the body will be 100 feet. above the ground, and ascending; when $t = \frac{70}{9.8}$ sec. the body will be 100 ft. above the ground, and descending. Therefore

(1) $$v = 49 - 9.8 \times \frac{28}{9.8} = 21 \text{ m. per sec.}$$

(2) $$v = 49 - 9.8 \times \frac{70}{9.8} = -21 \text{ m. per sec.}$$

17. *Two balloons start upwards together, one with a uniform velocity of* 8 *ft. per second, the other with a uniform acceleration of* 8 *ft. per second. How far apart will they be after* 1 *min.?*

The first balloon will rise in one minute a distance equal to $8 \times 60 = 480$ ft.

The second balloon will rise in one minute a distance equal to $\frac{1}{2} \times 8 \times 60^2 = 14,400$ ft.

At the end of one minute they will be $14,400 - 480 = 13,920$ ft. apart.

18. *How long will it take a body to slide down a smooth inclined plane* 100 *ft. long and* 25 *ft. high, and what velocity will it acquire?*

By § 172, we have, for the acceleration down the plane, $\frac{25}{100} \times 32 = 8$ ft. per sec. each second.

Hence $\quad 100 = \frac{1}{2} \times 8 \times t^2$, whence $t = 5$ sec.

$\therefore$ time required $= 5$ seconds.

Velocity acquired $= 5 \times 8 = 40$ ft. per sec.

19. *One body is allowed to slide down a smooth inclined plane* 800 *ft. long and* 100 *ft. high. Another body is allowed to fall vertically through the height of the plane. Find* (1) *the velocity of each body on reaching the base of the plane,* (2) *the time required for each body to fall.*

Let a denote the acceleration down the plane.

v the velocity acquired by the body on the plane.

v' the velocity acquired by the body falling vertically.

t the time required for the body to fall down plane.

t' the time required for the body to fall vertically.

Then $\quad a = \dfrac{100 \times 32}{800} = 4$ ft. per sec. each second.

$$(1)\quad \begin{cases} v^2 = 2as = 2 \times 4 \times 800, & \text{whence } v = 80 \text{ ft. per sec.} \\ v'^2 = 2 \times 32 \times 100, & \text{whence } v' = 80 \text{ ft. per sec.} \end{cases}$$

$$(2)\quad \begin{cases} t = \dfrac{80}{4} = 20 \text{ seconds.} \\ t' = \dfrac{80}{32} = 2.5 \text{ seconds.} \end{cases}$$

20. *Same as No.* 19, *only the length of the plane is* l, *and the height* h.

Using the same notation as in the last exercise,

$$a = \frac{gh}{l}, \text{ whence}$$

$$(1)\quad \begin{aligned} v^2 &= 2al = \frac{2ghl}{l}, \text{ whence } v = \sqrt{2gh}. \\ v'^2 &= 2gh, \text{ whence } v' = \sqrt{2gh}. \end{aligned}$$

Therefore, whatever be the length of the plane, the velocity acquired in descending the plane is the same as that acquired by a body in falling freely through a distance equal to the height of the plane.

$$(2)\qquad t = \frac{v}{a} = \frac{\sqrt{2gh}}{a}$$

$$t' = \frac{v'}{g} = \frac{\sqrt{2gh}}{g}$$

Hence $$t : t' = \frac{\sqrt{2gh}}{a} : \frac{\sqrt{2gh}}{g} = g : a = l : h.$$

21. *A body starts with the velocity 4, and has a uniform acceleration of 2. Find (1) the velocity after 10 seconds, (2) the mean velocity for this time, (3) the space described.*

(1) $v = u + gt \quad = 4 + 20 = 24$
(2) Mean velocity $= \frac{1}{2}(4 + 24) = 14$
(3) $s = 14 \times 10 \quad = 140.$

22. *A body has the velocity 30 and a uniform retardation of 3. Find (1) the velocity after 8 seconds, (2) the mean velocity for this time, (3) the space described, (4) the whole space described before coming to rest.*

(1) $v = u - gt \quad = 30 - 24 = 6$
(2) Mean velocity $= \frac{1}{2}(30 + 6) = 18$
(3) $s = 18 \times 8 \quad = 144$
(4) Whole space described $= 15 \times \frac{30}{3} = 150.$

23. *In 3 minutes after starting from a station a train is traveling at the rate of 40 miles an hour. Find (1) the mean acceleration for this interval in feet per second, and (2) the space described.*

Velocity acquired $= \dfrac{40 \times 5280}{3600}$ ft. per. sec.

Mean acceleration $= \dfrac{40 \times 5280}{3600 \times 180} = \dfrac{44}{135}$ ft. per sec.

Space described $= \dfrac{1}{2} \times \dfrac{44}{135} \times (180)^2 = 5280$ ft. $= 1$ mile.

24. *A train is moving at the rate of 45 miles per hour. On rounding a curve, the engineer sees another train a quarter of a mile ahead at rest on the track. He reverses his lever and puts on the brakes, thus causing a retardation of 3 ft. per second each second. Will the train stop in season to avoid a collision?*

The formula given at the bottom of page 184 may be applied to this case, if we substitute for "height ascended," "space described before coming to rest," and for "g" "a retardation of 3 ft. per second each second."

$$\text{Velocity in feet per second} = \frac{45 \times 5280}{3600}$$

$$\text{Space described before stopping} = \frac{v^2}{2g} = 726 \text{ ft.}$$

A quarter of a mile = 1320 ft.

Therefore the train will stop before collision.

25. *A car started down an incline of* 8 *per cent grade. Friction causes a retardation of* 2 *ft. per second each second. How far will the car move in* 2 *minutes?*

$$\text{Acceleration due to gravity} = \frac{8 \times 32}{100} = 2.56 \text{ ft. per sec. each sec.}$$

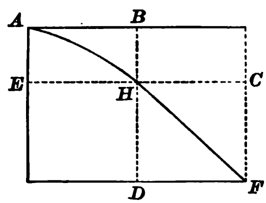
Retardation due to friction = 2 ft. per sec. each sec.

Actual acceleration = 0.56 ft. per sec. each sec.

Space described in 2 minutes = $\frac{1}{2} \times 0.56 \times (120)^2 = 4032$ ft.

26. *Show by a figure how to find the magnitude and direction of the velocity of the projectile in Fig.* 159 *after it has been moving* 1 *second.*

At the end of one second the projectile is at the point H, and its vertical velocity is equal to g. Draw HC to represent the constant horizontal velocity of the projectile, and HD vertical, and of such a length as to represent the vertical velocity g. Complete the parallelogram $HCFD$, and draw HF. HF represents in magnitude and direction the velocity of the projectile at the end of one second.



27. *With what velocity must a ball be fired horizontally from a point* 160 *ft. above the ground to have a horizontal range of* 2,000 *ft., the resistance of the air being left out of account?*

Let t denote the time of falling 160 ft.

Then $160 = 16t^2$, whence $t = \sqrt{10}$.

$$\text{Horizontal velocity required} = \frac{2000}{\sqrt{10}} = 200\sqrt{10} = 632.4 \text{ ft. per sec.}$$

28. *A cannon ball is fired at an angle of elevation of* 30° *with a velocity of* 1600 *ft. per second. Find* (1) *the time of flight,* (2) *the maximum height,* (3) *the horizontal range.*

Resolve the velocity into vertical and horizontal components. The vertical component = 800, the horizontal component = $800\sqrt{3}$. The time of ascent and the height ascended are the same as if the ball had been projected vertically with the velocity 800. During this time the ball also has a horizontal velocity of $800\sqrt{3}$.

(1) Time of flight $= 2 \times \frac{800}{32} = 50$ seconds.

(2) Maximum height $= \frac{800 \times 800}{2 \times 32} = 10{,}000$ ft.

(3) Horizontal range $= 800\sqrt{3} \times 50 = 40{,}000\sqrt{3}$ ft.

Class-room Exercises. Page 191.

1. *How is the head of a hammer often fastened when it gets loose, and how is the action which takes place explained?*

Move the hammer rapidly in the direction of the length of the handle, so that the motion shall be suddenly arrested by the free end of the handle coming in collision with a fixed object. The handle stops moving; but the head, by its inertia, moves on a little distance, and so becomes more firmly fixed upon the handle.

2. *How do you explain the fact that a circus rider can leap through a hoop by simply springing directly upwards?*

See § 174 (2).

3. *Why does a pendulum continue its swing against the action of gravity after the bob reaches the lowest point?*

The inertia of the pendulum tends to make its motion continue after passing the lowest point.

4. *Suspend a weight by a string (Fig. 162), and attach a piece of the same string to the weight underneath. If you pull the lower string with a sudden jerk, it breaks. But if you pull steadily, the upper string breaks. Explain.*

If the lower string is pulled with a sudden jerk, the ball, by its inertia, resists the tendency to a change of state so strongly that the string is broken. If the pull is steady and increased slowly, the upper string has to support this pull in addition to the weight of the ball, and very soon breaks.

5. *A bullet may be fired through a pane of glass and do no further damage than to leave a small round hole. But the bullet, if thrown by the hand against the glass, would shatter it in pieces. Account for this difference.*

When the bullet is fired through the glass, its motion is very rapid, and the glass with which the bullet does not come in contact is acted upon for so short a time that its inertia is not overcome. When the bullet is simply thrown against the glass, the bullet is moving so slowly that the time is sufficiently great for the bullet to overcome the inertia of the glass over a considerable extent of its surface.

6. *Why cannot a plate full of soup be quickly pushed across a table without spilling the soup?*

When the plate is pushed quickly across the table, the plate moves more quickly than the soup which it contains, and leaves some of this soup behind.

7. *A charge of dynamite placed on the top of a rock and exploded will shatter the rock. Can you account for this?*

When dynamite explodes, it is instantaneously turned into hot gases which exert an enormous pressure. The rock is shattered because the inertia of the air is not overcome as it would be in the case of a more slowly burning explosive.

8. *How does a hare, when pursued by a hound, make use of the law of inertia in order to escape from the hound?*

The hare suddenly darts to one side, but the inertia of the hound is so great that he cannot immediately change the direction of his motion, consequently the hare gains on the hound.

9. *Explain why you can throw a stone a greater distance by means of a sling than by your hand alone.*

With the sling we gradually give to the stone a great velocity of rotation which is suddenly changed into a velocity of translation, when the stone is released from the sling. With the hand alone, this great velocity cannot be given to the stone.

Class-room Exercises. Page 197.

Values of g; 32 with the foot as unit, and 980 with the cm. as unit.

1. *What must be the velocity of 1 ton that it may have the same momentum as a mass of 2 lb. moving with a velocity of 1,200 ft. per second?*

Let x denote the velocity required.

Then
$$2000\,x = 2 \times 1200$$
$$\therefore x = 1.2 \text{ ft. per sec.}$$

2. *The mass of an iceberg is 50,000 tons, and that of a steamboat is 200 tons. Their respective volocities are 6 ft. a minute and 15 miles an hour. Which has the greater momentum?*

Momentum of iceberg $= 50{,}000 \times 6 \times 60 = 18{,}000{,}000.$

Momentum of steamboat $= 200 \times 15 \times 5280 = 15{,}840{,}000.$

Hence the iceberg has the greater momentum.

3. *What is the momentum of a mass of 1 lb. after falling 1 second?*

Velocity at end of 1 sec. $= 32$ ft. per sec.

Momentum $= 1 \times 32 = 32.$

4. *What is the momentum of a mass of* 1 *lb. after falling* 400 *ft.?*

$v^2 = 2 \times 32 \times 4000$, whence $v = 160$ ft. per sec.

Momentum $= 1 \times 160 = 160$.

5. *Find the force in dynamical measure with which gravity pulls* (1) *a mass of* 4 *lb.*, (2) *a mass of* 4 *grams*, (3) *a mass of* 4 *kg.*

Force = mass × acceleration. See § 179. See also § 180.

Hence (1) $4 \times 32 = 128$ poundals.

(2) $4 \times 980 = 3{,}920$ dynes.

(3) $4 \times 1000 \times 980 = 3{,}920{,}000$ dynes.

6. *What acceleration will a force equal to a weight of* 6 *lb. impart to a mass of* 24 *lb.? How far will a mass move in* 3 *seconds?*

Force $= 6 \times 32 = 192$ poundals.

Acceleration $= \frac{192}{24} = 8$ ft. per sec. each sec.

Space $= \frac{1}{2} \times 8 \times 3^2 = 36$ ft.

7. *What force in grams weight will give to a mass of* 200 *grams a velocity of* 6 *m. in* 1 *second?*

$$\text{Force} = 200 \times 6 \times 100 = 120{,}000 \text{ dynes} = \frac{120000}{980} = 120.408 \text{ grams.}$$

8. *A mass of* 1 *ton under the action of a constant force describes* 160 *ft. from rest in* 8 *seconds. Find the force in pounds weight.*

Find the acceleration by the formula $s = \frac{1}{2} a t^2$.

Force = mass × acceleration. Then reduce poundals to pounds (§ 180).

Let a denote the acceleration.

Then $160 = \frac{1}{2} \times a \times 8^2$, whence $a = 5$ ft. per sec. each sec.

Force $= 2{,}000 \times 5 = 10{,}000$ poundals $= \frac{10000}{32} = 312.5$ lb.

9. *Masses of* 7 *lb. and* 9 *lb. are connected by a light string passing over a smooth pulley. Find* (1) *the velocity after* 2 *seconds*, (2) *the distance either mass describes*, (3) *the tension of the string.*

Moving force $= 9 - 7 = 2$ lb. $= 64$ poundals.

$$\text{Acceleration} = \frac{64}{7 + 9} = 4 \text{ ft. per sec. each sec.}$$

(1) Velocity after 2 sec. $= 4 \times 2 = 8$ ft. per sec.

(2) Space described $= \frac{1}{2} \times 4 \times 2^2 = 8$ ft.

(3) Let T denote the tension of the string in pounds. Acceleration of mass of 17 lb. $= 32\,(17 - T)$. But this acceleration is 4 ft. per sec. each sec. Hence $32\,(17 - T) = 4$, whence $T = 16\frac{7}{8}$ lb.

10. *Masses of* 30 *grams and* 40 *grams hang by a cord over a pulley. Find the space described in* 1 *second by each mass.*

Moving force $= 40 - 30 = 10$ grams $= 9{,}800$ dynes.

Acceleration $= \dfrac{9800}{30 + 40} = 140$ cm. per sec. each sec.

Space described in 1 sec. $= \frac{1}{2} \times 140 \times 1^2 = 70$ cm.

11. *Masses of 14 lb. and 18 lb. are joined by a string over a pulley. How far will they move in 3 seconds? What is the tension of the string?*

Moving force $= 18 - 14 = 4$ lb. $= 128$ poundals.

Acceleration $= \dfrac{128}{18 + 14} = 4$ ft. per sec. each sec.

Space described in 3 sec. $= \frac{1}{2} \times 4 \times 3^2 = 18$ ft.

Let T denote the tension in pounds.

Then $32\,(18 - T) = 4$, whence $T = 17\frac{7}{8}$ lb.

12. *Two masses of 48 grams and 50 grams are attached by a cord passing over a pulley. Starting from rest, each mass describes 10 cm. in 1 second. Find the value of g.*

Moving force $= 50 - 48 = 2$ grams $= 2\,g$ dynes.

Acceleration $= \dfrac{2\,g}{50 + 48} = \dfrac{2\,g}{98}$ cm. per sec. each sec.

Hence $10 = \dfrac{1}{2} \times \dfrac{2g}{98} \times 1^2$; $\therefore g = 980$ cm. per sec. each sec.

13. *How far on a smooth horizontal plane will a mass of 3 lb. move in 2 seconds, if the moving force is a mass of 1 lb. connected with the mass of 3 lb. by a cord and hanging vertically?*

Moving force $= 1$ lb. $= 32$ poundals.

Acceleration $= \dfrac{32}{3 + 1} = 8$ ft. per sec. each sec.

Space described in 2 sec. $= \frac{1}{2} \times 8 \times 2^2 = 16$ ft.

14. *How far will a mass of 48 lb. move on a horizonal plane in 1 minute, if acted on by a force equal to the weight of 12 lb., (1) supposing no friction, (2) supposing the coefficient of friction to be $\frac{1}{6}$?*

(1) Moving force $= 12$ lb $= 384$ poundals.

Acceleration $= \frac{384}{48} = 8$ ft. per sec. each sec.

Space described in 60 sec. $= \frac{1}{2} \times 8 \times 60^2 = 14{,}400$ ft.

(2) The friction $= 8$ lb.; therefore, the moving force $= 12 - 8$, or 4 lb. $= 4 \times 32$ poundals $= 128$ poundals.

Acceleration $= \frac{128}{48} = \frac{8}{3}$ ft. per sec. each sec.

Space described in 60 sec. $= \frac{1}{2} \times \frac{8}{3} \times 60^2 = 4{,}800$ ft.

15. *In what time will 4 lb. hanging vertically draw 12 lb. through 45 feet on a smooth horizontal plane (1) supposing no friction, and (2) supposing the coefficient of friction to be $\frac{1}{4}$?*

(1) Moving force $= 4$ lb. $= 128$ poundals.

$$\text{Acceleration} = \frac{128}{12 + 4} = 8 \text{ ft. per sec. each sec.}$$

If t denotes the required time in seconds, we have

$$45 = \tfrac{1}{2} \times 8 \times t^2\,;\ \therefore\ t = \tfrac{3}{2}\sqrt{5} \text{ sec.}$$

(2) Moving force $= 4 - 12 \times \frac{1}{4} = 1$ lb. $= 32$ poundals.

$$\text{Acceleration} = \frac{32}{12 + 4} = 2 \text{ ft. per sec. each sec.}$$

If t denotes the required time in seconds, we have,

$$45 = \tfrac{1}{2} \times 2 \times t^2\,;\ \therefore\ t = 3\sqrt{5} \text{ sec.}$$

16. *How far will a mass of* 60 *kg. move on a horizontal plane in* 10 *seconds, if acted on by a force equal to the weight of* 30 *kg.*, (1) *supposing no friction*, (2) *supposing the coefficient of friction to be* $\frac{1}{3}$?

(1) Moving force $= 30$ kg. $= 30{,}000$ grams $= 29{,}400{,}000$ dynes.

$$\text{Acceleration} = \frac{294{,}000{,}000}{60 \times 1000} = 490 \text{ cm. per sec. each sec.}$$

Space described in 10 sec. $= \frac{1}{2} \times 490 \times 10^2 = 24{,}500$ cm.

(2) Moving force $= 30 - 60 \times \frac{1}{3} = 10$ kg. $= 10{,}000$ grams $= 9{,}800{,}000$ dynes.

$$\text{Acceleration} = \frac{9{,}800{,}000}{60 \times 1000} = 163\tfrac{1}{3} \text{ cm.}$$

17. *A train whose mass is* 200 *tons, moving at the rate of* 30 *miles an hour, is brought to rest in* 20 *seconds. What is the average value of the brake power? How far does the train move before coming to rest?*

30 miles an hour $= 44$ ft. per second.

Momentum $= 200 \times 2000 \times 44$. Let $x =$ force of the brakes.

Impulse of this force $=$ the momentum destroyed.

Or, $20\,x = 200 \times 2000 \times 44$, whence $x = 880{,}000$ poundals $= 27{,}500$ lb.

$$\text{Retardation} = \frac{\text{force}}{\text{mass}} = \frac{880{,}000}{200 \times 2000} = \tfrac{11}{5} \text{ ft. per second each second.}$$

Space described $= \frac{1}{2}\,a\,t^2 = \frac{1}{2} \times \frac{11}{5} \times 20^2 = 440$ ft.

18. *A train of* 640 *tons is moving at the rate of* 30 *miles an hour when the steam is shut off, and a brake power equal to the weight of* 20 *tons is applied. In what time will the train be brought to rest, and how far will it move before stopping?*

In this question it is simpler to take 1 ton as the gravitation unit of force.

Velocity of train $= 30$ miles per hour $= 44$ ft. per second.

Momentum of train $= 640 \times 44$.

Let t denote the time of stopping in seconds.
Impulse of retarding force $= 20 \times 32 \times t$.
Therefore (§ 183) $20 \times 32 \times t = 640 \times 44$, whence $t = 44$ sec.

$$\text{Retardation} = \frac{\text{retarding force}}{\text{mass}} = \frac{20 \times 32}{640}.$$

$$\text{Space} = \frac{1}{2} \times \frac{20 \times 32}{640} \times (44)^2 = 968 \text{ ft.}$$

19. *A* 1,000-*lb. shot strikes a target with a velocity of* 1,600 *ft. a second. How far will the shot penetrate if the target exert upon the shot an average pressure of* 12,000 *tons?*

Momentum of shot $= 1000 \times 1600$.

$$\text{Time of stopping} = \frac{\text{momentum}}{\text{force}} = \frac{1}{480} \text{ second.}$$

$$\text{Retardation} = \frac{12000 \times 2000 \times 32}{1000} = 768{,}000 \text{ ft. per sec. each sec.}$$

$$\text{Distance shot penetrates} = \tfrac{1}{2} \times 768{,}000 \times \left(\frac{1}{480}\right)^2 = 1\tfrac{2}{3} \text{ ft.}$$

20. *A weight of* 2 *tons drops on the head of a pile from a height of* 32 *ft., and drives the pile through a distance of* 1 *ft. What is the average resistance of the ground, the weight of the pile being neglected?*

Velocity of weight $= \sqrt{2gs} = \sqrt{2 \times 32 \times 32} = 32\sqrt{2}$ ft. per sec.
Momentum of weight $= 2 \times 32\sqrt{2} = 64\sqrt{2}$.
Let x denote (in tons) the average resistance of the ground.

$$\text{Retardation} = \frac{32x}{2} = 16x \text{ ft. per sec. each second.}$$

$$\text{Time of stopping} = \frac{32\sqrt{2}}{16x} = \frac{2\sqrt{2}}{x} \text{ seconds.}$$

Distance the pile is driven is 1 ft.

$$\therefore 1 = \tfrac{1}{2} \times 16x \times \left(\frac{2\sqrt{2}}{x}\right)^2, \text{ whence } x = 64 \text{ tons.}$$

21. *A train whose mass is* 200 *tons is drawn up an incline of* 4 *per cent grade at uniform speed. The resistance of friction amounts to* 8 *lb. per ton. Find the tractive force of the engine.*

Since there is no acceleration, the tractive force of the engine is equal to the resistance overcome. This resistance is composed of the component of the weight which acts down the incline and the force of friction.

Component acting down incline $= 200 \times 2000 \times 0.04 = 16{,}000$ lb.
Force of friction $= 200 \times 8 = 1{,}600$ lb.
Therefore required force $= 16{,}000 + 1600 = 17{,}600$ lb.

22. *What force will stop in half a mile a train of* 300 *tons moving at the rate of* 45 *miles an hour?*

Velocity of train $= \dfrac{45 \times 5280}{3600} = 66$ ft. per second.

Space described in stopping $= \frac{1}{2} \times 5280 = 2640$ ft.

Let a denote the retardation.

Since $v^2 = 2as$, $a = \dfrac{v^2}{2s} = \dfrac{66 \times 66}{2 \times 2640} = \dfrac{33}{40}$ ft. per sec. each sec.

$$\begin{aligned}\text{Retarding force} &= \text{mass} \times \text{retardation}\\ &= 300 \times 2000 \times \tfrac{33}{40} \text{ poundals}\\ &= \frac{300 \times 2000 \times 33}{32 \times 40} \text{ lb.} = 15{,}468 \text{ lb.}\end{aligned}$$

23. *What pressure will a man weighing* 160 *lb. exert on the floor of an elevator which is ascending with an acceleration of* 4 *ft. per second?*

The force required to give the man an acceleration of 4 ft. per second $= 160 \times 4$ poundals $= 20$ lb. The pressure = his weight + 20 lb. = 180 lb.

24. *A balloon descends with a uniform acceleration of* 8 *ft. per sec. What pressure will a man weighing* 200 *lb. exert on the floor of the car?*

Force required to give the man an acceleration of 8 ft. per sec. each second $= 200 \times 8 = 1600$ poundals.

1600 poundals $= \frac{1600}{32} = 50$ lb. Therefore,

the pressure of the man on the floor of the car $= 200 - 50 = 150$ lb.

Class-Room Exercises. Page 204.

1. *A* 24*-lb. ball leaves a gun whose mass is* 5 *tons with a velocity of* 1,200 *ft. per second. Find the velocity of recoil of the gun.*

Let v denote the velocity of recoil of the gun.

Then, as momentum of gun is equal to mementum of ball, we have

$$5 \times 2000 \times v = 24 \times 1200 \;;\; \therefore v = 2.88 \text{ ft. per sec.}$$

2. *A* 64*-lb. shot is fired from a gun whose mass is* 8 *tons; the velocity of recoil of the gun is* 6 *ft. per second. Find the velocity of the shot.*

Let v denote the velocity of the shot.

Then $64 \times v = 8 \times 2000 \times 6$; $\therefore v = 1500$ ft. per sec.

3. *A mass of* 10 *lb., moving with a velocity of* 50 *ft. per second, strikes a mass of* 50 *lb. at rest. If the masses keep together after impact, what is their common velocity?*

Let v denote the common velocity.

Total momentum before impact $= 10 \times 50 + 50 \times 0$

Total momentum after impact $= (10 + 50) \times v$

Hence $60v = 500$; $\therefore v = 8.33$ ft. per sec.

4. *Two inelastic masses, $A = 8$ lb., $B = 12$ lb., collide. Find the common velocity after collision if the velocities before collision are*

(1) *A 10 ft. per second, B 5 ft. per second in the same direction.*
(2) *A 10 ft. per second, B 5 ft. per second in opposite directions.*
(3) *A 10 ft. per second, B 20 ft. per second in opposite directions.*
(4) *A 10 ft. per second, B at rest.*

In each case let v denote the common velocity.

(1) Total momentum before collision $= 8 \times 10 + 12 \times 5 = 140$.
Total momentum after collision $= (8 + 12) \times v = 20\,v$.
Hence $20\,v = 140$; therefore $v = 7$ ft. per sec.

(2) Total momentum before collision $= 8 \times 10 - 12 \times 5 = 20$.
Total momentum after collision $= (8 + 12) \times v = 20\,v$.
Hence $20\,v = 20$; therefore $v = 1$ ft. per sec.

(3) Total momentum before collision $= 8 \times 10 - 12 \times 20 = -160$.
Total momentum after collision $= (8 + 12) \times v = 20\,v$.
Hence $20\,v = -160$; therefore $v = -8$ ft. per sec.

The minus sign means that the velocity is in the opposite direction to that of A before the collision.

(4) Total momentum before collision $= 8 \times 10 + 12 \times v = 80$.
Total momentum after collision $= (8 + 12) \times v = 20\,v$.
Hence $20\,v = 80$; therefore $v = 4$ ft. per sec.

5. *Two inelastic bodies, moving in opposite directions, collide, and are brought to rest. The velocities of the bodies before collision were* 48 *m. and* 100 *m. per second. The first body has a mass of* 150 *kg. What is the mass of the other body?*

Let m denote the required mass.

Total momentum before collision $= 150 \times 48 - 100 \times m$.
Total momentum after collision $= 0$
Hence $7200 - 100\,m = 0$; $\therefore m = 72$ kg.

6. *A body A with a mass* 8 *and velocity* 10 *overtakes a body B with mass* 12 *and velocity* 5. *After impact, B's velocity is* 9 *in the same direction as before. What is A's velocity?*

Let x denote A's velocity after impact.

Total momentum before impact $= 8 \times 10 + 12 \times 5 = 140$
Total momentum after impact $= 8 \times x + 12 \times 9 = 8\,x + 108$
Hence $8\,x + 108 = 140$; $\therefore x = 4$.

7. *A perfectly elastic mass of* 120 *kg., moving at the rate of* 286 *m. per second, collides with a similar mass of* 45 *kg. which is at rest. Find the velocities after collision.*

Let v denote the velocity of the mass of 120 kg. after collision, and v' the velocity of the mass of 45 kg.

Total momentum before collision $= 120 \times 286 + 45 \times 0 = 34320$

Total momentum after collision $= 120 \times v + 45 \times v'$

Hence $120\,v + 45\,v' = 34320$ (1)

By [2], page 201, $v' - v = 286 - 0$ (2)

Multiplying (2) by 45 and subtracting from (1), we have

$$166\,v = 34{,}320 - 286 \times 45 = 21{,}450$$

$$\therefore v = 129.217 \text{ m. per sec.}$$

$$v' = 286 + 129.217 = 415.217 \text{ m. per sec.}$$

8. *A fishing boat weighing 4 tons is 40 yards from the shore. A man in it hauls a cask weighing 200 lb. from the shore. How far will the cask be from the shore when it reaches the boat?*

Let x denote the distance of the cask from the shore.

Then $40 - x =$ distance through which the boat moves.

Mass of cask = 200 lb. Mass of boat = 8000 lb.

$$\therefore 200\,x = 8000\,(40 - x)$$

$$x = 40\,(40 - x)$$

$$41\,x = 1600, \text{ whence } x = 39\tfrac{1}{41} \text{ yds.}$$

9. *A hand-car weighing 720 lb. and running 10 ft. per second collides with another weighing 640 lb. and running in the opposite direction at the rate of 15 ft. per second. What will be the result?*

Assume that the two bodies move on together after the collision. Let x denote the common velocity.

Total momentum before collision $= 720 \times 10 - 640 \times 15 = -2400.$

Total momentum after collision $= 1360\,x.$

$\therefore 1360\,x = -2400$, whence $x = -1\tfrac{13}{17}$ ft. per second.

The two bodies move after collision in the direction of the motion of the larger hand-car, and at the rate of $1\tfrac{13}{17}$ ft. per second.

10. *A shell weighing 200 lb. and moving at the rate of 1,000 ft. per second explodes into two parts, one of which weighs 50 lb. and is just brought to rest. What is the velocity of the other part?*

Let x denote the velocity of the moving part.

Total momentum before collision $= 200 \times 1000.$

Total momentum after collision $= 50 \times 0 + 150\,x.$

$\therefore 150\,x = 200 \times 1000$, whence $x = 1333\tfrac{1}{3}$ ft. per sec.

11. *An ounce bullet is fired into a block of wood at rest, which weighs 4 pounds. If the block by the impact receives a velocity of 16 ft. per second, find the velocity of the bullet before impact.*

Let x denote velocity of the bullet before impact.

Total momentum before impact $= \frac{x}{16}$

Total momentum after impact $= (4 + \frac{1}{16})\ 16$

$\therefore \frac{x}{16} = (4 + \frac{1}{16})\ 16$, whence $x = 1040$ ft. per sec.

Class-room Exercises. Page 209.

1. *Find the time of vibration of a simple pendulum* 49 *cm. long at a place where* $g = 980$ *cm. per second each second.*

By the formula, § 190,

$$t = 3.1416 \times \sqrt{\frac{49}{980}} = 0.7025 \text{ sec.}$$

2. *Find the length in centimeters of a pendulum vibrating once a second at a place where* $g = 980$.

By the formula, § 190,

$$1 = 3.1416 \times \sqrt{\frac{l}{980}}$$

Whence $$l = \frac{980}{3.1416^2} = 99.32 \text{ cm.}$$

3. *Find the length of a seconds pendulum (a pendulum that vibrates in one second) at New York where* $g = 32.16$.

Let l denote the required length.

Then $$1 = 3.1416 \times \sqrt{\frac{l}{32.16}}$$

Hence, $$l = \frac{32.16}{3.1416^2} = 3.26 \text{ ft.}$$

4. *Compare the lengths of four pendulums that oscillate in* 1, 2, 3, *and* 4 *seconds, respectively.*

By law 3, § 191, the lengths are as the numbers 1, 4, 9, and 16.

5. *Assuming a seconds pendulum to be* 1 *meter long, what will be the length of a pendulum that vibrates in* 10 *seconds, and the length of a pendulum that vibrates in half a second?*

Let x denote the length of the pendulum that vibrates in 10 seconds.

Then $1 : 10 = \sqrt{1} : \sqrt{x}$; $\therefore x = 100$ meters.

Let y denote the length of the pendulum that vibrates in half a second.

Then $1 : \frac{1}{2} = \sqrt{1} : \sqrt{y}$; $\therefore y = \frac{1}{4}$ meter, or 25 cm.

6. *If a certain pendulum vibrate once a second, what is the time of vibration of a pendulum* 9 *times as long? Also, what is the time of vibration of a pendulum half as long?*

Let x denote the time of vibration for a pendulum 9 times as long.

Then $$1 : x = \sqrt{1} : \sqrt{9}\ ;\ \therefore\ x = 3 \text{ sec.}$$

Let y denote the time of vibration for a pendulum half as long.

Then $$1 : y = \sqrt{1} : \sqrt{\tfrac{1}{2}}\ ;\ \therefore\ y = \sqrt{\tfrac{1}{2}} = \tfrac{1}{2}\sqrt{2} = 0.707 \text{ sec.}$$

7. *If an iron ball suspended by a fine wire from the cupola of St. Paul's in London make* 176 *vibrations in half an hour, what is the height of the cupola above the floor?*

Let x denote the required height in feet.

$$\text{Time of 1 vibration} = \frac{\frac{1}{2} \times 60 \times 60}{176} = \frac{225}{22} \text{ sec.}$$

Then $$\frac{225}{22} = 3.1416 \times \sqrt{\frac{x}{32.19}}$$

$$x = \frac{225^2 \times 32.19}{22^2 \times 3.1416^2} = 341.2 \text{ ft.}$$

8. *How will the time of vibration of a pendulum be affected by taking it to the top of a high mountain?*

At the top of a high mountain the value of g would be less than at the base of the mountain; consequently, if we examine the formula, $t = \pi\sqrt{\frac{l}{g}}$, we see that t, the time of vibration, would be increased. For as g is made smaller, the quantity under the square root sign must become larger.

9. *If a seconds pendulum were taken to a place where the force of gravity is only one-fourth of that at the surface of the earth, in what time would the pendulum make one vibration? If the pendulum were attached to a clock, how much would the clock appear to lose in* 24 *hr.?*

If t and t' denote the times of vibration of a pendulum at places where the values of g are g and g' respectively,

$$t = \pi\sqrt{\frac{l}{g}} \text{ and } t' = \pi\sqrt{\frac{l}{g'}},$$

whence $$t : t' = \pi\sqrt{\frac{l}{g}} : \pi\sqrt{\frac{l}{g'}} = \frac{1}{\sqrt{g}} : \frac{1}{\sqrt{g'}} = \sqrt{g'} : \sqrt{g}.$$

If $g' = \frac{1}{4}g$, we have $t : t' = \frac{1}{2}\sqrt{g} : \sqrt{g} = \frac{1}{2} : 1$; whence $t' = 2t$.

$\therefore$ the pendulum would make one vibration in 2 seconds.

If the pendulum were attached to a clock, and we compared the rate of the clock with that of a good chronometer, which is driven by a spring and is therefore independent of the force of gravity, we should observe that while the chronometer counted out for us two hours, the

clock only indicated a period of one hour. Therefore 24 hours by the chronometer would be only 12 hours by the clock; that is, the clock would appear to lose 12 hours.

10. *What is meant by the length of a compound pendulum? How is the length found by experiment?*

By the length of a compound pendulum is meant the distance from the axis to the center of oscillation.

To find the length of a compound pendulum, the time of vibration is accurately determined. Then the pendulum is supported at a point supposed to be near its center of oscillation, and the time of vibration is again carefully taken. If this time of vibration is not the same as before, the position of the point of support is changed till the time of vibration is equal to that at first obtained. The distance between the two points of suspension is the length of the compound pendulum. See § 192.

Class-room Exercises. Page 213.

1. *Give a clear explanation of the meaning of the terms "centripetal force" and "centrifugal force," and illustrate by an example.*

See § 176 and Fig. 161. The pull of the hand in the direction SC is called the *centripetal* force, and its effect is to cause a continual change in the direction of the motion of the body S. The reaction of S to this change, equal but opposite in direction to the pull of the hand, is called *centrifugal* force. If the string breaks, both action and reaction cease to exist. The string is then free to obey the First Law of Motion, and moves off in the direction of the tangent ST.

2. *A stone weighing* 10 *lb. is attached to a cord* 3 *ft. long, and whirled around twice a second. Find the tension of the string.*

Circumference of the circle $= 6\pi$ ft.

Velocity of the stone $= 12\pi$ ft. per second.

Tension of string $= \dfrac{mv^2}{r} = \dfrac{10 \times 144\pi^2}{3} = 4800$ poundals $= 150$ lb.

3. *A mass of* 5 *lb. is compelled to move in a circle of radius* 16 *ft., at the rate of* 900 *ft. per minute. Find the centripetal force.*

$$\text{Centripetal force} = \frac{5 \times (\frac{900}{60})^2}{16} = 70.3125 \text{ poundals} = 2.197 \text{ lb.}$$

4. *A mass of* 10 *lb. at the end of a wire* 4 *ft. long revolves* 100 *times a minute. Find the tension of the wire.*

Circumference of circle $= 8\pi$ ft.

Velocity of mass $= \frac{100 \times 8\pi}{60} = \frac{40\pi}{3}$ ft. per sec.

Tension of wire $= \frac{10 \times \left(\frac{40\pi}{3}\right)^2}{4} = 4390.02$ poundals $= 137.19$ lb.

5. *A car whose mass is 4 tons and velocity 20 miles an hour goes round a curve of 1100 ft. radius. Find the pressure on the rails.*

$$\text{Pressure on rails} = \frac{4 \times 2000 \times \left(\frac{20 \times 5280}{60 \times 60}\right)^2}{1100} = 6257.77 \text{ poundals}$$
$$= 195.55 \text{ lb.}$$

6. *How many times a minute must a mass of 1 lb. revolve horizontally at the end of a string 1 ft. long in order that the tension of the string may be equal to the weight of 1 lb.?*

Let v denote the velocity of the moving mass.

By substitution in the formula $F, = \frac{mv^2}{r}$, we have $v^2 = 32$;

Whence $v = 4\sqrt{2} = 5.656$ ft. per sec.
Distance passed over in 1 minute $= 5.656 \times 60$
Circumference of circle $= 2\pi$ ft.

Number of revolutions per minute $= \frac{5.656 \times 60}{2\pi} = 53.99$.

7. *Taking the earth's radius as 4,000 miles, and $\pi^2 = 10$, find by how much the weight of a body at the equator is diminished by the rotation of the earth on its axis.*

$$\text{Acceleration} = a = \frac{4\pi^2 r}{t^2} = \frac{4 \times 10 \times 4000 \times 5280}{(24 \times 60 \times 60)^2} = \frac{55}{486}.$$

Therefore, $a : g = \frac{55}{486} : 32$; that is $\frac{a}{g} = \frac{1}{282}$ nearly.

Hence the weight of a body at the equator is diminished by about $\frac{1}{282}$ of its true weight.

Review Exercises on Chapter V. Page 216.

1. *Define uniform motion and velocity.*

A point is said to be in *uniform motion* when it passes over equal spaces in equal times.

The *velocity* of a moving point is the rate at which its position is changing.

2. *How is the resultant of two component velocities at right angles to each other found?*

By the Parallelogram Law. See § 162.

3. *Show that rest and motion are relative terms.*

See § 164.

4. *Define motion of translation and motion of rotation. What sort of motion has a point on the tire of a carriage wheel?*

See § 165. A point on the tire of a carriage wheel has motion of rotation about the axis, as well as motion of translation due to the forward movement of the axis.

5. *How is variable velocity measured? How is the mean velocity of a moving point for any interval of time found?*

See § 166 and § 167.

6. *What is meant by acceleration? How is it measured?*

See § 168.

7. *Explain the meaning of the formulas,* $v = gt$, $s = \frac{1}{2} g t^2$, $v^2 = 2 gs$; *and show how they are obtained.*

See § 169.

8. *Prove that acceleration down an inclined plane is found by multiplying g by the ratio of the height of the plane to its length.*

See § 172.

9. *Prove that a body thrown vertically upwards with the velocity u will rise to the height of* $\frac{u^2}{2g}$.

See § 173 (2).

10. *Show by a diagram that the path of a projectile is a curve.*

See § 173 (3).

11. *State Newton's First Law of Motion, and give examples.*

See § 174.

12. *Apply the First Law to the motion of a body in a circle.*

See § 176.

13. *State Newton's Second Law. What is the measure of a force?*

See § 179. Force is measured by the rate of change of momentum which it produces.

14. *Define the poundal and the dyne. How are forces reduced from gravitation measure to absolute measure, and vice versa?*

See § 180.

15. *What is the dynamical definition of equal masses?*

Two bodies have equal masses if the same force will generate in them the same velocity in the same time. See § 181.

16. *Define the impulse of a force, and show that it is equal to the total momentum produced by the force.*

See § 196.

17. *Give an example of the physical independence of forces.*

See § 184.

A cannon ball fired horizontally from the top of a tower. The force of the powder causes the ball to describe a certain number of feet each second in a horizontal direction. The force of gravity causes the ball to fall vertically with an accelerated motion, such that the space described during the first second is 16 ft., during the next second 64 ft., during the third second 144 ft., etc. Each of these forces produces exactly the same effect on the ball as if the other force did not exist. The true position of the ball at any time is found by applying the parallelogram law. See § 173 (3).

18. *State and illustrate Newton's Third Law.*

See § 185.

19. *Prove the formula for the collision of two inelastic bodies.*

See § 186.

20. *Prove that when two perfectly elastic bodies collide the difference of the velocities is unaltered by the collision.*

See § 186, page 201.

21. *What laws result from the application of Newton's Third Law to internal forces?*

See § 188.

22. *State the laws of the motion of a pendulum.*

See § 191.

23. *What is meant by the center of oscillation of a compound pendulum?*

See § 192.

24. *Prove the formula for acceleration in circular motion.*

See § 194.

25. *State the Law of Universal Gravitation. Mention some of the phenomena which Newton explained by means of it.*

See § 197 and § 198.

CHAPTER VI.

ENERGY.

Class-room Exercises. Page 221.

1. *How much work is done in raising* 6 *cubic feet of water to a height of* 8 *ft.?*

$$6 \times 62.5 \times 8 = 3{,}000 \text{ ft.-lb.}$$

2. *How high must* 7 *lb. be raised in order to do* 84 *ft.-lb. of work?*

Let x denote the height in feet.

Then $$7x = 84, \text{ whence } x = 12 \text{ ft.}$$

3. *The ram of a pile driver weighs* 784 *lb., and has a fall of* 23 *ft. How much work must be expended in raising the ram?*

$$784 \times 23 = 18{,}032 \text{ ft.-lb.}$$

4. *In what time will a man pump* 400 *cubic feet of water to the mean height of* 30 *ft., allowing that he can perform* 2,600 *ft.-lb. of work per minute, and neglecting friction?*

Let x denote the number of hours required.

Then $400 \times 62.5 \times 30 = 2600 \times x \times 60$.

$13x = 62.5$; $\therefore x = 4\frac{21}{26}$ hours, or 4 hours 48.46 minutes.

5. *If a pit* 20 *ft. deep and* 8 *sq. ft. in section is excavated, how much work is done, a cubic foot of earth weighing* 100 *lb.?*

Weight of earth excavated $= 20 \times 8 \times 100 = 16{,}000$ lb. The C. G. of this earth is raised 10 ft. $\therefore$ work done $= 160{,}000$ ft.-lb.

6. *By pumping* 3 *tons of water out of a well, the distance to the water level is increased from* 20 *ft. to* 26 *ft. Find how much work is done.*

The C. G. of the 3 tons of water is raised $\dfrac{20+26}{2} = 23$ ft.

Hence the work done $= 3 \times 2000 \times 23 = 138{,}000$ ft.-lb.

7. *A mass of* 200 *lb. is pushed* 40 *ft. up an incline of* 10 *per cent grade. The friction amounts to a force of* 16 *lb. How much work is done against gravity? How much against friction?*

By § 203, Case 2, we have

Work done against gravity $= 200 \times \frac{40}{100} \times 40 = 800$ ft.-lb.
Work done against friction $= 40 \times 16 = 640$ ft.-lb.

8. *A man weighing* 140 *lb. puts a load of* 140 *lb. on his back, and carries it up a ladder to the height of* 50 *ft. What amount of work does he do altogether, and how much of this work is done usefully?*

$$\text{Total amount of work} = 280 \times 50 = 14{,}000 \text{ ft.-lb.}$$

The amount of work done usefully is that performed in carrying the load of 140 lb. up the ladder.

$$\text{This work is equal to } 140 \times 50 = 7{,}000 \text{ ft.-lb.}$$

9. *The area of the piston of a steam engine is* 2,000 *sq. in., the mean effective pressure of the steam is* 30 *lb. per sq. in., the length of the stroke is* 4 *ft., and the number of complete strokes per minute is* 40. *How much work does the engine do in* 1 *minute?*

$$\text{Work done in 1 minute} = 2000 \times 30 \times 4 \times 40 \times 2 = 19{,}200{,}000 \text{ ft.-lb.}$$

NOTE. — A complete stroke is the movement of the piston from end to end of the cylinder and back again.

10. *A bicyclist makes* 15 *miles an hour on a level road. He exerts a downward pressure of* 20 *lb. with each foot during the down stroke, and the length of the stroke is* 1 *ft. The diameter of the driving wheel is* 28 *inches. Find the work he does per minute.*

$$\text{No. of feet passed over in 1 minute} = \frac{15 \times 5280}{60} = 1320.$$

$$\text{No. of feet passed over in 1 revolution} = \frac{2 \times 14 \times 22}{12 \times 7} = \frac{22}{3}$$

$$\text{No. of revolutions in 1 minute} = \frac{1320}{\frac{22}{3}} = 180.$$

$$\text{Work done during 1 revolution} = 2 \times 20 = 40 \text{ ft. lb.}$$

$$\text{Work done per minute} = 180 \times 40 = 7200 \text{ ft.-lb.}$$

11. *What is the horse power of an engine that raises* 60 *cubic feet of water per minute from a depth of* 242 *ft.?*

$$\text{The work done} = 60 \times 62\tfrac{1}{2} \times 242 \text{ ft.-lb.}$$

$$\text{Therefore, the horse power} = \frac{60 \times 125 \times 242}{2 \times 33000} = 27.5.$$

12. *Find the horse power of an engine that will raise* 14 *tons of coal* (2,240 *lb. each) in* 1 *hour from a pit whose depth is* 480 *ft.*

$$\text{Work done per hour} = 480 \times 14 \times 2240 \text{ ft.-lb.}$$

$$\text{Therefore, the horse power} = \frac{480 \times 14 \times 2240}{60 \times 33000} = 7.6.$$

13. *A forge hammer weighs* 484 *lb. and rises once a second, each lift being* 3 *ft. What is the horse power of the engine that works it?*

Work done in 1 minute $= 484 \times 60 \times 3$ ft. lb.

Therefore, the horse power $= \dfrac{484 \times 60 \times 3}{33000} = 2.64.$

14. *Find the horse power of an engine capable of raising* 30 *tons through a height of* 44 *ft. in* 5 *minutes.*

Work done in 1 minute $= \dfrac{30 \times 2000 \times 44}{5} = 528{,}000$ ft.-lb.

Therefore, the horse power $= \dfrac{528{,}000}{33{,}000} = 16.$

15. *A windmill raises by means of a pump* 15 *tons of water per hour to a height of* 66 *ft. Calculate its horse power.*

Work done in 1 minute $= \dfrac{15 \times 2000 \times 66}{60}$

Therefore, the horse power $= \dfrac{15 \times 2000 \times 66}{60 \times 33000} = 1.$

16. *How many tons of coal* (2,240 *lb. each*) *will an engine of* 3 *H. P. raise per hour from a coal pit whose depth is* 720 *ft.?*

A 3 H. P. engine will do $3 \times 60 \times 33{,}000$ ft.-lb. of work per hour.

Work in raising 1 ton of coal 720 ft. $= 2240 \times 720$

$\therefore$ the number of tons raised $= \dfrac{3 \times 60 \times 33000}{2240 \times 720} = 3.68.$

17. *From what depth will an engine of* 20 *H. P. pump* 40 *cubic feet of water in* 1 *minute?*

Let x denote the depth in feet.

Then work done in 1 minute $= 62.5 \times 40 \times x.$

The horse power $= \dfrac{40 \times 62.5 \times x}{33000} = 20.$

Hence $2.5x = 660$, whence $x = 264$ ft.

18. *In what time will an engine of* 40 *effective H. P. pump* 4,000 *cubic feet of water to a mean height of* 360 *ft.?*

Let x denote the time in minutes.

Then work done in 1 minute $= \dfrac{4000 \times 62.5 \times 360}{x}.$

The horse power $= \dfrac{4000 \times 62.5 \times 360}{x \times 33{,}000} = 40.$

Hence $11x = 750$, $x = 68\frac{2}{11}$ minutes, or 1 hour, 8 minutes nearly.

19. *If a man does* 1,000,000 *ft.-lb. of work in a working day of* 8 *hr., with what fraction of a horse power does he work on the average?*

The work done in 1 minute $= \dfrac{1{,}000{,}000}{8 \times 60}$,

Therefore, the horse power $= \dfrac{1{,}000{,}000}{8 \times 60 \times 33000} = \dfrac{25}{396} = 0.063.$

20. *A vertical shaft* 440 *ft. deep and* 10 *sq. ft. in cross-section is full of water. What is the horse power of an engine that will empty it in* 6 *hours, if friction consumes one-third of the power of the engine?*

Let x denote the required horse power.

The average height through which the water is raised $= \frac{440}{2} = 220$ ft.

Volume of water raised $= 440 \times 10$ cu. ft.

Work done in 1 minute $= \dfrac{4400 \times 62.5 \times 220}{6 \times 60}$ ft. lb.

As friction consumes one-third of the power of the engine, the effective power is $\dfrac{2x}{3}$.

Therefore $\dfrac{2x}{3} = \dfrac{4400 \times 62.5 \times 220}{6 \times 60 \times 33000}$.

Hence $18x = 137.5$, and $x = 7.64$.

21. *What is the horse power of an engine that draws a train of* 132 *tons at the uniform rate of* 45 *miles an hour against a resistance amounting to* 900 *lb.?*

Let x denote the required horse power.

The work done in 1 minute $= \dfrac{900 \times 45 \times 5280}{60}$ ft. lb.

Therefore $x = \dfrac{900 \times 45 \times 5280}{60 \times 33000} = 108$ H. P.

22. *At what rate is an engine working when it drives a train of* 300 *tons at the rate of* 15 *miles an hour, the resistance to motion being equal to* 22 *lb. per ton?*

Let x denote the required horse power.

Work done in 1 minute $= \dfrac{300 \times 22 \times 15 \times 5280}{60}$ ft. lb.

Therefore $x = \dfrac{300 \times 22 \times 15 \times 5280}{60 \times 33000} = 264$ H. P.

23. *A steam engine supplies* 1,000 *houses each with* 22 *cubic feet of water daily, the water being raised to a mean height of* 100 *ft. If the engine works twelve hours a day, what is its rate of working?*

Let x denote the required horse power.

Work done in 1 minute $= \dfrac{22 \times 62.5 \times 1000 \times 100}{12 \times 60}$ ft.-lb.

Therefore $x = \dfrac{22 \times 62.5 \times 1000 \times 100}{12 \times 60 \times 33000} = 5.77$ H. P.

24. *Find the amount of horse power transmitted by a belt passing over a wheel 7 ft. in diameter which makes 3 revolutions a second, the tension of the belt being equal to 88 lb.*

Let x denote the required horse power.

Work done in 1 minute $= 88 \times 3 \times 60 \times 7 \times \frac{22}{7}$.

Therefore $x = \dfrac{88 \times 3 \times 60 \times 22}{33000} = 10.56$ H. P.

25. *The area of the piston of a steam engine is 500 sq. in., the mean pressure of the steam is 33 lb. per square inch, the length of the stroke is 4 ft., and the number of full strokes per minute is 32. Find the horse power of the engine.*

Let x denote the required horse power.

Work done in 1 minute $= 33 \times 500 \times 4 \times 2 \times 32$.

Therefore $x = \dfrac{33 \times 500 \times 4 \times 2 \times 32}{33000} = 128$ H. P.

26. *A shaft 560 ft. deep and 5 ft. in diameter is full of water. How long would it take a $3\frac{1}{2}$ H. P. engine to pump out the water, friction being neglected?*

Let x denote the number of hours required.

Then work done in 1 minute $= \dfrac{560 \times (\frac{5}{2})^2 \times \frac{22}{7} \times 62.5 \times \frac{560}{2}}{60\,x}$ ft.-lb.

Therefore, the horse power $= \dfrac{560 \times \frac{25}{4} \times \frac{22}{7} \times 62.5 \times 280}{60 \times x \times 33000} = 3\frac{1}{2}$.

$9\,x = 250$, whence $x = 27$ hours $46\frac{2}{3}$ minutes.

27. *A water-power engine of 10 H. P. is supplied from a tank 12 ft. high, 8 ft. long, 6 ft. wide, at a height of 120 ft. Supposing the tank to be full, but no supply, find how long the engine could run.*

Whole power supplied $= 12 \times 8 \times 6 \times 62.5 \times 120$ ft.-lb.

Power consumed by engine per minute $= 10 \times 33000$ ft.-lb.

Time of running $= \dfrac{12 \times 8 \times 6 \times 62.5 \times 120}{10 \times 33000} = 13$ min. nearly.

28. *A steam crane, working with 6 H. P., raises 20 tons to a height of 150 ft. in an hour. How much of the work is done against friction?*

The engine can do $6 \times 33000 \times 60$ ft.-lb. work per hour.

Useful work done per hour $= 20 \times 2000 \times 150$ ft.-lb.

Waste work = total work − useful work = 5,880,000 ft.-lb.

29. *If it requires a 2 H. P. engine to raise 200 cubic feet of water per hour through 80 ft., what per cent of the work is waste work?*

Total work done by engine in 1 minute $= 2 \times 33000$ ft.-lb.

Work done in 1 min. by engine in raising water $= \dfrac{200 \times 62.5 \times 80}{60}$ ft.-lb.

Waste work $= 66000 - 16666\frac{2}{3} = 49333\frac{1}{3}$ ft.-lb.

Class-room Exercises. Page 233.

1. *The handle of a claw hammer is 15 in. long, and the claw is 3 in. long. When a force of 50 lb. is applied to the handle, the nail is drawn out. What is the resistance of the nail?*

Let x denote the resistance of the nail. By § 207,

$$3x = 15 \times 50$$
$$x = 250 \text{ lb.}$$

2. *A weight of 240 kg. is raised 20 cm. by means of a movable pulley. How much force is required? How far does the force move? How much work is done?*

By § 212, the force required is 120 kg.

The force moves twice as far as the weight, or 40 cm.

$$\text{Work done} = \frac{240 \times 20}{100} = 48 \text{ kilogram-meters.}$$

3. *The radius of a wheel is 80 cm., and the radius of the axle is 12 cm. What weight can be supported by a force of 30 kg.? How much work is done if the weight is raised 60 cm.?*

By § 211, we have, if x denotes the required weight,

$$12x = 80 \times 30, \text{ whence } x = 200 \text{ kg.}$$

$$\text{Work done} = \frac{30 \times 60}{100} = 18 \text{ kilogram-meters.}$$

4. *What must be the ratio of the radii of a wheel and axle in order that a force of 100 lb. may just support 1 ton?*

By § 211, we have $$\frac{r}{r'} = \frac{2000}{100} = 20.$$

The radius of the wheel must be 20 times that of the axle.

5. *A pair of pulley blocks contain each four pulleys. The rope is attached to the upper fixed block. What force is just sufficient to raise a weight of 1,600 lb. if the friction amounts to a force of 50 lb.?*

Let x denote the required force.

The number of parts of the cord at the lower block is 8. By § 212,

$$\frac{1600}{x-50} = 8.$$
$$1600 = 8x - 400$$
$$8x = 2000\text{ , whence } x = 250 \text{ lb.}$$

6. *An inclined plane is 50 ft. long and 30 ft. high. What force is required to make a mass of 80 lb. ascend the plane, and how much work against gravity is done in the ascent, (1) if there is no friction, (2) if the coefficient of friction between the body and the plane is $\frac{1}{4}$?*

(1) Let x denote the required force. By § 207, we have

$$50x = 30 \times 80\text{ , } x = 48 \text{ lb.}$$

Work done against gravity $= 80 \times 30 = 2400$ ft.-lb.

(2) Let y denote the perpendicular pressure on the inclined plane.

Then $\quad 50 : 40 = 80 : y\text{ , } y = 64$

Force of friction $\quad = 64 \times \frac{1}{4} = 16$ lb.

Hence force required $\quad = 48 + 16 = 64$ lb.

The amount of work done against gravity is the same as in (1).

7. *A force of 75 lb. is exerted along a smooth inclined plane 5 yd. long and 3 ft. high. What weight will it raise?*

Let x denote the weight in pounds.

By the Principle of Work,

$$3x = 75 \times 5 \times 3\text{ , whence } x = 375 \text{ lb.}$$

8. *A man can push with a force of 200 lb., and wishes to raise a body weighing 1,000 lb. into a cart 3 ft. high. How long an inclined plane must he employ?*

Let x denote the length of the inclined plane.

By the Principle of Work,

$$200x = 1000 \times 3\text{ , whence } x = 15 \text{ ft.}$$

9. *How great a weight can a force of 27 lb., applied horizontally on a smooth inclined plane 26 ft. long, raise to the top of a wall 10 ft. high?*

Let x denote the weight in pounds.

By the Principle of Work,

$$10x = 27\sqrt{26^2 - 10^2} = 27\sqrt{(26+10)(26-10)}$$
$$= 27\sqrt{36 \times 16} = 27 \times 6 \times 4$$
$$x = 64.8 \text{ lb.}$$

10. *The lever of a screw is 3 ft. 6 in. long, and the distance between two threads is $\frac{1}{2}$ of an inch. What pressure must be applied to the end of the lever to produce a pressure of 10 tons upon the press board?*

Let x denote the force required.

By the Principle of Work,

$$2 \times \tfrac{7}{2} \times \tfrac{22}{7} \times x = \frac{10 \times 2000}{5 \times 12}.$$

$$22x = \frac{1000}{3}, \text{ whence } x = 15.15 \text{ lb.}$$

11. *The lever of a screw is 1 ft. 9 in. long, and the power applied at the end is 100 lb. What must be the thickness of the threads that a pressure of 5,000 lb. may act upon the press board?*

Let x denote the pitch of the screw in inches.

By the Principle of Work,

$$2 \times \tfrac{7}{4} \times \tfrac{22}{7} \times 100 = \frac{5000x}{12}.$$

$$50x = 132, \; x = 2.64 \text{ in.}$$

12. *What pressure is exerted by a force of 1 lb. acting at a distance of 4 ft. 8 in. from the axis of a screw, if the distance between the threads is 1 in.?*

Let x denote the required pressure.

By the Principle of Work,

$$2 \times 4\tfrac{2}{3} \times \tfrac{22}{7} \times 1 = \frac{x}{12}.$$

$$x = 352 \text{ lb.}$$

13. *A loaded car weighing 16 tons is to be drawn up an incline of 10 per cent grade by means of a rope coiled around an axle 2 ft. in diameter, the diameter of the wheel being 10 ft. Find the force that must be applied to the wheel.*

Let x denote the required force in pounds.

Mechanical advantage of wheel and axle $= \tfrac{10}{2} = 5$.
Mechanical advantage of inclined plane $= \tfrac{100}{10} = 10$.
Total mechanical advantage $= 5 \times 10 = 50$.

That is, a force of 1 lb. applied to the rim of the wheel will balance a weight of 50 lb. on the incline.

Hence $$x = \frac{16 \times 2000}{50} = 640 \text{ lb.}$$

Or we may apply the Principle of Work directly.

Work of x during one revolution of wheel $= 10 \times \tfrac{22}{7} \times x$.

When the wheel makes one revolution, the weight moves up the plane a distance of $2 \times \tfrac{22}{7}$ ft. Since the grade is 10%, the weight is raised against gravity a distance of $\dfrac{2 \times \frac{22}{7}}{10}$ ft.

$$\therefore 10 \times \tfrac{22}{7} \times x = \frac{2 \times 22 \times 32000}{7 \times 10},$$

whence $$x = 640, \text{ as before.}$$

Class-room Exercises. Page 241.

1. *How much kinetic energy has a cannon ball weighing* 64 *lb. and moving with a velocity of* 1,600 *ft. per second?*

By § 217, we have

$$\text{Kinetic energy of ball} = \frac{64 \times 1600^2}{2 \times 32} = 2{,}560{,}000 \text{ ft.-lb.}$$

2. *A mass of* 4 *lb. falls from rest to the ground in* 3 *seconds. How much kinetic energy does it have on reaching the ground? What becomes of this energy when the mass comes to rest?*

By Law 1, § 217,

$$\text{Kinetic energy} = \frac{4 \times V^2}{2 \times 32}.$$

$$\text{But} \qquad V = 32 \times 3 = 96.$$

$$\therefore \text{Kinetic energy} = \frac{4 \times 96^2}{2 \times 32} = 576 \text{ ft.-lb.}$$

When the mass strikes the ground, its energy of motion is used in making an indentation in the ground, in causing sound and a rise in temperature, and in setting the ground into vibration.

3. *A ball weighing* 1 *lb. is fired vertically upwards with a velocity of* 160 *ft. a second. Compute its kinetic and its potential energy at the start and at the end of each second during the time that it ascends.*

$$\text{Kinetic energy at starting} = \frac{1 \times \overline{160}^2}{2 \times 32} = 400 \text{ ft.-lb.}$$

$$\text{Time of ascent} \qquad = \tfrac{160}{32} = 5 \text{ seconds.}$$

The following table gives the velocities, heights, the kinetic energy, and the potential energy, after each second has elapsed.

TIME.	VELOCITY.	HEIGHT IN FEET.	K. E. IN FT.-LB.	P. E. IN FT.-LB.	SUM.
0	160	0	400	0	400
1	128	144	256	144	400
2	96	256	144	256	400
3	64	336	64	336	400
4	32	384	16	384	400
5	0	400	0	400	400

This example illustrates the general principle that the sum of the kinetic and the potential energy at any instant is constant, and equal to

the kinetic energy at the start, or to the potential energy at the highest point reached.

4. *A mass of* 32 *lb. falls from rest to the ground, a distance of* 400 *ft. What is its kinetic energy on reaching the ground?*

$$\text{Velocity on reaching the ground} = \sqrt{2 \times 32 \times 400} = 160 \text{ ft. per sec.}$$

$$\text{Kinetic energy} = \frac{32 \times 160^2}{2 \times 32} = 12{,}800 \text{ ft.-lb.}$$

5. *How many ft.-lb. of energy does a ton of water possess at the height of* 1 *mile above the sea level? What kind of energy is it?*

$$\text{Potential energy} = 2000 \times 5280 = 10{,}560{,}000 \text{ ft.-lb.}$$

6. *A stream of water is flowing at the rate of* 4 *miles per hour. Find the momentum and the kinetic energy per cubic foot of water.*

$$\text{Velocity of stream} = \frac{4 \times 5280}{60 \times 60} = \frac{88}{15} \text{ ft. per sec.}$$

$$\text{Momentum} = 62\tfrac{1}{2} \times \tfrac{88}{15} = 366.67$$

$$\text{Kinetic energy} = \frac{\frac{125}{2} \times (\frac{88}{15})^2}{2 \times 32} = 33.61 \text{ ft.-lb.}$$

7. *Describe the transformations of energy that take place in the swinging of a pendulum.*

When the pendulum starts to swing down the arc, all its energy is potential. As it swings down the arc, it constantly gains kinetic energy at the expense of potential energy till it reaches the lowest point of its swing, when all its energy is kinetic. As the pendulum continues its journey and ascends the arc, it gains potential energy at the expense of kinetic energy till it reaches the highest point of its swing, when all its energy is potential again. For every swing of the pendulum these transformations of energy take place.

8. *A mass of* 80 *lb., moving with a velocity of* 16 *ft. a second, strikes an equal mass at rest. Both masses are inelastic, and move on together after the impact. Find the total kinetic energy of the masses,* (1) *before the impact,* (2) *after the impact. Explain the apparent loss of energy.*

$$(1) \text{ Kinetic energy} = \frac{80 \times 16^2}{2} = 10{,}240 \text{ ft. poundals.}$$

$$(2) \text{ Kinetic energy} = \frac{160 \times v^2}{2}$$

$$\text{But} \quad 80 \times 16 = 160 \times v \;;\; \therefore v = 8 \text{ ft. per sec.}$$

$$\therefore \text{Kinetic energy} = \frac{160 \times 8^2}{2} = 5120 \text{ ft. poundals.}$$

The mass has been doubled, while the velocity has been halved. Doubling the mass tends to make the kinetic energy twice as great,

halving the velocity tends to make the kinetic energy one-quarter as great, since the kinetic energy of a body varies directly as its mass, and also varies directly as the square of its velocity. Thus, on the whole, the kinetic energy has been halved. But the loss of energy is only apparent. Some has been expended in altering the shape of the colliding masses, and some has been transformed into heat. See § 223.

9. *A mass of 100 grams, moving with a velocity of 24 cm. a second, overtakes and adheres to a mass of 60 grams, moving in the same direction with a velocity of 8 cm. per second. Find the common velocity and the total kinetic energy (in gram-centimeters) before and after the impact.*

Let v denote the common velocity.

$$\text{Momentum before impact} = 100 \times 24 + 60 \times 8$$

$$\text{Momentum after impact} = (100 + 60)\,v$$

$$160\,v = 2880\text{ , whence } v = 18\text{ cm. per sec.}$$

$$\text{Kinetic energy before impact} = \frac{100 \times 24^2}{2 \times 980} + \frac{60 \times 8^2}{2 \times 980} = 31.35$$

$$\text{Kinetic energy after impact} = \frac{160 \times 18^2}{2 \times 980} = 26.45.$$

10. *A mass of 60 lb., moving at the rate of 10 ft. a second, is acted upon by a force directly opposite to the motion until the mass has a velocity of 30 ft. a second in the direction of the force. How much kinetic energy has the force imparted to the mass?*

The force will bring the mass to rest with an expenditure of energy

$$= \frac{60 \times 10^2}{2 \times 32}\text{ ft.-lb.}$$

To give the mass a velocity of 30 ft. per sec., the amount of energy required

$$= \frac{60 \times 30^2}{2 \times 32}\text{ ft.-lb.}$$

$$\text{Hence the total amount of energy} = \frac{60}{2 \times 32}(100 + 900) = 937.5\text{ ft.-lb.}$$

11. *A body whose mass is 3 lb. makes 42 revolutions per second in a circle whose radius is 14 ft. How much kinetic energy does it have?*

$$\text{Velocity of mass} = 2 \times 14 \times \tfrac{22}{7} \times 42 = 12{,}936\text{ ft. per sec.}$$

$$\text{Kinetic energy} = \frac{3 \times 12936^2}{2 \times 32} = 7{,}844{,}067\text{ ft.-lb.}$$

12. *A mass of 8 lb., starting from rest, is acted upon by a force of 2 lb. for 30 seconds. How much kinetic energy does it acquire?*

If v denotes the velocity acquired, we have

$$v = \frac{2 \times 32 \times 30}{8} = 240 \text{ ft. per sec.} \quad (\text{See § 182.})$$

$$\text{Kinetic energy} = \frac{8 \times 240^2}{2 \times 32} = 7{,}200 \text{ ft.-lb.}$$

13. *A body whose mass is* 10 *lb. is moving at the rate of* 24 *ft. a second, when a constant resistance equal to* 2 *oz. begins to act directly against the motion. How far will the body move before it comes to rest?*

$$\text{Kinetic energy} = \frac{10 \times 24^2}{2 \times 32} = 90 \text{ ft.-lb.}$$

If s denotes the distance through which the body moves before it comes to rest, we shall have

$$\tfrac{2}{16} \times s = 90 \,, \; s = 720 \text{ ft.}$$

14. *A train whose mass is* 120 *tons is moving at the rate of* 45 *miles an hour, when the steam is shut off, and a brake power equal to* 4,000 *lb. is applied. How far will the train go before coming to rest?*

$$45 \text{ miles per hour} = \frac{45 \times 5280}{3600} = 66 \text{ ft. per sec.}$$

$$\text{Kinetic energy} = \frac{120 \times 2000 \times 66 \times 66}{2 \times 32} \text{ ft.-lb.}$$

Let x denote the distance through which the train moves before coming to rest.

$$4000\, x = \frac{120 \times 2000 \times 66 \times 66}{2 \times 32}$$

$$x = 4083.75 \text{ ft.}$$

Review Exercises on Chapter VI. Page 261.

1. *Define work, and the common unit of work; also give a numerical example to illustrate the formula, work = force × distance.*

Work is the act of changing the position of a body by overcoming resistance to the change. (See § 199.) The *foot-pound* is the work done in raising a weight of one pound through a distance of one foot. For example, see § 200.

2. *How is the work done by a force which acts obliquely on a moving body estimated?*

The work done by a force acting obliquely on a moving body is measured by the product of the force and the distance through which the body moves in the direction of the force. See § 203.

3. *Name and define the common unit employed to measure rate of working.*

The *horse power* is the ability to do 33,000 foot-pounds of work per minute. See § 204.

4. *Define the foot-poundal and the erg.*

The *foot-poundal* is the work done by a poundal in moving its point of application one foot in the direction in which it acts. See § 205.

An *erg* is the work done by a dyne in moving its point of application one centimeter in the direction in which it acts. See § 205.

5. *What is the use of a machine? Illustrate your answer by some special case.*

A machine is a contrivance by means of which a force may be applied to perform a definite kind of work advantageously. See § 206.

In the wheel and axle (Fig. 177) a weight Q is raised by means of a smaller weight P.

If one arm of a lever is 10 times as long as the other arm, a force acting at the end of the long arm will balance 10 times as great a force acting at the end of the short arm.

6. *State the Principle of Work. Show by an example the meaning of the statement that what is gained in power is lost in speed.*

The *Principle of Work* is that the amount of work done by a machine is equal to the work applied to the machine. See § 207.

For an illustration that "what is gained in force is lost in speed," see § 212.

7. *Define the terms efficiency and mechanical advantage, and illustrate by an example.*

The efficiency of a machine is the ratio of the useful work to the whole work applied. See § 209.

The mechanical advantage of a machine is the ratio of the weight to the power.

For illustration, see § 209.

8. *Apply the principle of work to the wheel and axle.*

See § 211.

9. *Apply the principle of work to the screw.*

See § 214.

10. *What is the simplest way of finding the mechanical advantage of a compound machine?*

To find the mechanical advantage of a compound machine, find the ratio of the distance through which the power moves to the distance through which the weight moves. See § 215.

11. *Prove the formula for kinetic energy given in* § 217.

See § 217.

12. *Explain why a rifle ball will bore a hole through a two-inch plank while the recoil of the gun only gives the shoulder a slight shock.*

See § 218.

13. *Give examples of bodies possessing potential energy.*

A bent bow, a coiled spring, the weight of a clock when the clock is wound up.

14. *How is potential energy in the case of gravity measured?*

In the case of gravity, the potential energy of a body is measured by the product of its weight and its distance above the surface of the earth. See § 221.

15. *Give examples of the generation of heat by work.*

See illustrations, § 223.

16. *Give examples of the conversion of heat into mechanical energy.*

See illustrations, § 224.

17. *What is the true nature of heat?*

See § 225.

18. *Explain the statement that the mechanical equivalent of heat is 1400 ft.-lb.*

See § 226.

19. *Explain the statement that the calorific power of carbon is* 8,000 *calories.*

See § 229.

20. *State Carnot's Principle.*

The maximum efficiency of a heat engine is independent of the material substance employed to carry heat into the engine, and depends only on the temperatures at which the heat enters and leaves the engine. See § 230.

21. *Explain with diagrams how the heat of steam is converted into mechanical energy by a steam engine.*

See Figs. 192 and 193, p. 251.

22. *Give an example of energy undergoing transformations.*

See § 234.

23. *State the Law of Conservation of Energy. What are some of the grounds for believing it to be true?*

Energy can assume a great variety of forms, but cannot be created or destroyed. See § 235.

24. *Name some of the sources of energy, and show that the sun is the main source.*

See §§ 236, 237.

25. *What is the general tendency of potential energy of every kind? What does it compel us to believe?*

Potential energy tends to assume the form of visible kinetic energy, and finally the invisible form of uniformly diffused heat. This leads us to think that energy is undergoing a process of degeneration in quality considered with reference to its ability to do work. See § 239, 2.

CHAPTER VII.

MAGNETISM AND ELECTRICITY.

Laboratory Experiments. Page 269.

1. *Perform such experiments as suggest themselves to you, with a magnet and some iron nails for the purpose of illustrating magnetic induction* (§ 245).

It is interesting to insert a sheet of paper between the nail and the magnet to show that the magnetic induction will take place through paper as well as through air.

2. *Magnetize by contact a steel knitting needle* (§ 247) *; then separate it into parts, and examine the magnetic properties of each part.*

In separating the needle into parts, break by bending with the hands; do not strike the needle with a hammer, because sudden jars may weaken the strength of a magnet.

3. *Perform the experiment with a glass tube full of iron filings which is described in* § 248.

Be sure that the walls of the tube and the iron filings are dry.

4. *With the aid of iron filings make a map of the lines of force around a bar magnet* (§ 249).

The filings may be fixed in their positions by applying a thin coating of shellac varnish to the paper beforehand, and then directing a gentle current of steam upon the paper after the filings have arranged themselves into lines of force. When the varnish hardens, the filings stick to the paper.

Do not get too many filings on the paper. When sprinkling the filings on the paper, let them fall some distance, so that they may be spread evenly.

5. *Make a map of the lines of force around and between two bar magnets placed parallel to each other.*

When the like poles of the two magnets are adjacent to each other, the general character of the lines of force is indicated in Fig. 1. When

the unlike poles are adjacent to each other, the general character of the lines of force is indicated in Fig. 2.

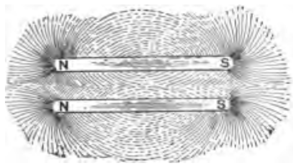


FIG. 1.

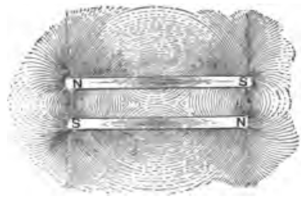


FIG. 2.

Class-room Exercises. Page 270.

1. *Why does a magnetic needle, floating on water, point towards magnetic north and south, but not move bodily in either direction?*

See § 250.

2. *If you have a bar magnet and a knitting needle, one end of which is notched with a file, describe how you would proceed in order to convert the needle into a magnet with its north pole at the notched end.*

Put the knitting needle on the table, place the south pole of the bar magnet on the unnotched end of the needle, and draw it along the entire length of needle. Repeat this operation 15 or 20 times. The needle will now be magnetized with its north pole at the notched end.

If the poles of the bar magnet are not marked, suspend the magnet by a thread, and find its polarity by observing how it sets.

3. *Two sewing needles are magnetized so that their eyes are north poles. They are then stuck through bits of cork and thrown into water, so that they float with their eyes upward. How will they behave towards each other, and how if the south pole of a magnet is held over them?*

The two upper poles are alike, and also the two lower. Therefore the needles will move away from each other (§ 243).

If the south pole of a *strong* magnet is held over the needles, they will be attracted; if the distance is not too great, the needles will move towards the south pole of the magnet until their north poles are in contact with it. If the magnet is weak, it must be held very near one of the needles before the attraction causes any motion.

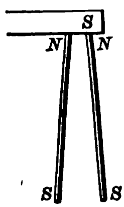
4. *How can you tell whether a steel rod is magnetized or not,* (1) *by means of a compass needle,* (2) *without the aid of a magnet or of iron?*

(1) Bring one end of the rod towards the north pole of the compass needle, and observe the effect. Bring the same end of the rod towards

the south pole of the compass needle, and observe the effect. If the rod is neutral, there will be attraction in both cases. If it is magnetized, there will be attraction in one case, and repulsion in the other.

(2) Divide the rod into two parts. Suspend one part horizontally by a thread, and then bring the same end of the other part in succession towards the ends of the suspended part. If there is attraction in one case, and repulsion in the other, the rod is magnetic. If no effect is observed, the rod is neutral. (See § 248.)

5. *Two iron rods hang from the same pole of a magnet. Will they hang parallel? Give a sketch and reasons for your answer.*



The lower ends of the rods are converted by induction into like magnetic poles. Therefore they will repel each other, and the rods will not be parallel, but assume a position like that shown in the Figure. (See § 245.)

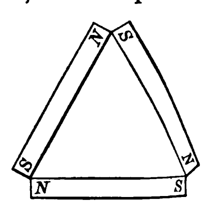
6. *You have two similar rods, one of steel, the other of soft iron. You have also a bar magnet and some iron nails. Describe how you would tell which was the steel and which the iron rod.*

Magnetize both the rods as explained in § 247. After a short time, test the magnetism of the rods by means of the iron nails as shown in Fig. 199. The steel rod will be found to be a much stronger magnet than the iron rod.

7. *A bar magnet is laid on a table with its north pole projecting over the edge. An iron nail clings to the projecting end. State and explain what happens when the south pole of another magnet is brought above and near the north pole of the first.*

The nail is held to the magnet by induction. When the south pole of the second magnet is brought near the north pole of the first magnet, the two opposite poles nearly neutralize each other, so far as power to act on other bodies is concerned. Therefore the inductive action of the magnet on the nail becomes so weak that it cannot counterbalance the force of gravity, and the nail falls.

8. *You have three equal bar magnets without keepers. How would you arrange them so that when not in use they will best retain their magnetism? Give a sketch.*



Arrange the three magnets so as to form a triangle, with unlike poles in contact in each case.

9. *If a long bar of very soft iron is held upright, why does its lower end repel the north pole of a compass needle?*

When the bar is held upright, it is acted upon by the earth's magnetic field of force, which tends in the northern hemisphere to move a north pole downwards. Therefore the lower end of the bar becomes by induction a north pole, and will repel a similar pole.

10. *The beam of a balance is made of soft iron. When it is placed at right angles to the magnetic meridian, two equal weights placed in the pans just balance. Will the weights still appear to be equal when the beam is placed in the magnetic meridian? Explain.*

When the beam is placed in the magnetic meridian the end towards the north will fall, and the other end will rise (provided the mass of soft iron is large enough, and the suspension delicate enough). For the beam will be magnetized by the inductive action of the earth, and will tend to set itself parallel to the direction of the terrestrial lines of magnetic force which in the latitude of New York are inclined some 70° to the horizon.

11. *If a compass needle were carried around the equator, would it point in the same direction at all places? If not, state as nearly as you can what changes would be observed in its behavior during its journey.*

The compass needle would not point in the same direction at all places. If the needle is carried on the equator westward, starting from the meridian of Greenwich, its declination, at first westerly, will diminish, and become zero before reaching America. The declination will then become easterly, and after reaching a maximum value, will diminish to zero again. From this point the declination is westerly to the end of the journey.

12. *A tall iron mast stands just in front of the compass on a wooden ship. What effect will be produced in the declination of the compass when the ship is sailing due east,* (1) *in the northern hemisphere,* (2) *in the southern hemisphere?*

(1) In the northern hemisphere the bottom of the mast will be a north pole. This pole will be to the east of the compass, and repel its north pole. Therefore the declination of the compass, if westerly, will be made greater than its true value, and if easterly, less than its true value.

(2) In the southern hemisphere, the lower end of the mast will be a south pole, and the north pole of the compass needle will be drawn to the east of its true position, producing effects the reverse of those mentioned under case (1).

Laboratory Exercises. Page 284.

NOTE. — All experiments with statical electricity should be performed when the air is dry. Bright, cold days in winter are the best for these experiments.

Devise, perform, and describe an experiment to prove that:

1. *Two bodies if rubbed together become electrified with opposite charges.*
2. *Brass is a conductor and glass a non-conductor of electricity.*
3. *A moist thread conducts electricity better than dry thread.*
4. *There is no electricity on the inside of a charged conductor.*
5. *Electricity escapes at points faster than from a flat surface.*

1. Suspend two bits of cork (or two pith balls) by dry, silk threads. Place a flannel cap to which a silk thread is attached over one end of a rod of sealing wax, both cap and sealing wax being previously warmed. Rub the cap round the wax several times. Remove the cap by the silk thread held in one hand, and hold the rod by the unrubbed end in the other hand. Bring the cap near one of the suspended corks, and the rod near the other. The bits of cork will be attracted till they touch the cap and the rod respectively; and then, being charged by contact, they will be repelled. Now bring the bits of cork very near each other. They will be attracted. This proves that the cap and the rod are electrified with opposite charges (§ 256).

2. Hold a brass rod in the hand, rub it with silk, and then bring it near a suspended piece of cork. The cork is unaffected. Perform the same experiment with a glass rod. The cork is attracted. Wrap some silk (a non-conductor) round the end of the brass rod, and repeat the experiment with the rod. The cork is attracted. We infer that electricity remains on the glass where it is generated, but will readily pass from the brass to the hand which is in direct contact with it. In other words, glass is a non-conductor, but brass a conductor, of electricity.

3. Suspend a bit of cork by a perfectly dry, silk thread, and bring near it a glass rod previously rubbed with silk, or a stick of sealing wax previously rubbed with flannel. Observe the effect. Wet the silk thread, and repeat the experiment. Observe the effect. The difference in the effects proves that moist thread is a better conductor than dry thread.

4. Suspend a bit of cork by a dry, silk thread. Fasten with mucilage or sealing wax a small empty tin can (with the lid removed) to a plate of glass. Charge the can strongly by means of an electrophorus. Then test the can for an electric charge by holding the suspended cork near the tin, first on the outside of the can, and then on the inside.

5. Use the electroscope described in the next exercise. Fit a bullet in which a small hole has been drilled over the pointed end of the brass rod of the electroscope. Charge the leaves of the electroscope. Observe the time required for the leaves to collapse. Recharge the leaves of the electroscope with a charge of the same amount as before, and remove the bullet from the end of the brass rod. Observe again the time required for the leaves to collapse. Instead of a bullet, it is better to use a hollow brass ball with a small hole of such size that the ball can be fitted to the end of the brass rod.

6. *Construct a gold-leaf electroscope, and describe how you would use it to determine whether a body had a positive or a negative charge.*

Take a glass bottle with a wide mouth. See that the bottle, inside and out, is perfectly dry and free from dust. Procure a cork which will fit tightly into the mouth of the bottle. Punch a hole through the middle of the cork, and push through the hole a brass rod of such length that when its upper end projects 3 or 4 in. above the cork, its lower end shall be about 3 in. above the bottom of the bottle. Bend half an inch of the lower end so as to form a right angle. File the upper end to a point. Place a strip of gold leaf 4 in. long and $\frac{1}{2}$ inch wide over the bend in the rod so that two pieces of gold leaf each 2 in. long hang side by side. Fasten the gold leaf to the rod with sealing wax, taking care not to interrupt the metallic connection between the rod and the gold leaf. Fit upon the upper end of the rod a bullet in which a hole of the proper size has been drilled, or better, a hollow brass ball having a hole of the proper size. Insert the cork in the bottle. The electroscope is now complete and ready for use.

For the method of determining whether a body has a positive or a negative charge, see § 261.

Class-room Exercises. Page 284.

1. *How would you proceed in order to electrify a glass rod given to you? How would you prove that it is electrified? How would you ascertain whether the charge is positive or negative?*

A rod is electrified by rubbing it vigorously with fur, wool, or silk. The rod then becomes capable of attracting light bodies, such as cork or pith balls (§ 252). To ascertain the kind of charge, charge a pith ball positively, as explained in § 255, then bring the electrified rod near the pith ball; if there is attraction, the rod has a negative charge; if there is repulsion, the rod is positively charged (§ 256).

2. *Describe a gold-leaf electroscope. How would you charge it positively, and how negatively?*

See § 261.

3. *You have a rod of unknown material. How would you determine whether it is a good or a bad conductor of electricity?*

Try to electrify the rod by friction. If the rod, after being well rubbed, becomes electrified, and remains in this state for some time, you may infer that the material of the rod is a bad conductor. If the rod shows no sign of electrification the inference is that the material is a good conductor. The electricity, as fast as it is generated, flows along the rod, and through the hand and body to the earth.

4. *An electroscope is charged negatively, and an insulated brass ball is brought near the disc. What conclusion as to the electrical state of the ball do you come to* (1) *when the leaves slightly collapse,* (2) *when they slightly diverge?*

(1) The brass ball is either unelectrified or has a positive charge. (2) The brass ball has a negative charge (§ 261).

5. *How can a series of sparks be obtained from an electrophorus?*

Charge the disc of the electrophorus as explained in § 258, then hold the disc very near an uncharged piece of metal or the knob of a Leyden jar. A spark will cross the air gap between the two bodies. Then repeat the operation.

6. *Describe a Leyden jar, the method of charging it, and the method of discharging it.*

See § 267.

7. *Explain the action of a condenser of electricity.*

See § 266.

8. *One person holds a charged Leyden jar in his hand by its outer coating, and another person holds similarly an uncharged jar. What will happen when the knobs of the two jars are brought together?*

When the knobs of the two jars are brought very near, a spark will be seen, and a part of the charge in the charged jar will pass to the uncharged jar. At the same time each person will receive a shock, owing to the passage of a certain amount of electricity from the outer coating of the jar which he is holding to the ground. The shock will be less severe than that which the person holding the charged jar would suffer if he were to apply his hand to the knob of the jar.

9. *A brass rod held in the hand and beaten with catskin shows no electricity when it is made to touch an electroscope. How would you prove that it was really electrified when so beaten?*

If the brass rod is provided with a glass handle, and then beaten, it will be found to be electrified. When the rod is held in the hand, the electricity flows through the hand to earth as fast as it is generated.

10. *If a cent is fastened to the end of a stick of sealing wax, how could you give it a negative charge by the help of a positively charged rod?*

Take the end of the sealing wax in the hand, and hold the cent near the positively charged rod. Touch the cent while in this position with the finger, take the finger away, and then withdraw the cent. The latter is now charged negatively by induction (§ 257).

11. *A copper vessel is insulated and electrified. If touched at different parts with a proof plane, what part will show the strongest charge, and what part the weakest charge?*

The edges of the copper vessel will show the strongest charge; the flattest parts of the vessel will show the weakest charge (§ 264).

12. *Why do you get no shock on touching the knob of a charged Leyden jar which stands on a vulcanite plate?*

Because vulcanite is an excellent insulator. When you touch the knob of the jar with your finger, the charge on the inner coating cannot pass through the vulcanite plate and unite with the charge on the outer coating. Therefore no shock is felt.

13. *Why is it a surer test that a body is electrified, if it repels an electrified pith ball suspended by a silk thread than if it attracts the ball?*

Attraction leaves it doubtful whether the body is unelectrified or has a charge unlike that of the pith ball (see § 256). Repulsion takes place only when *both* bodies are electrified with like charges. Therefore repulsion is a surer test of electrification.

14. *A piece of dry brown paper, laid on a warm metal tray, is rubbed with catskin. The tray is then placed on a dry glass tumbler, and the brown paper is removed. Why can you now get a spark on bringing your knuckle near the tray?*

The rubbing electrifies the brown paper negatively. The charge on the paper acts inductively on the metal tray, repelling negative electricity to earth, and attracting positive electricity to the upper surface of the tray. When the tray is placed on the tumbler, both paper and tray retain their charges. When the paper is removed, the influence of its charge upon the tray is destroyed, and the positive charge on the tray is free to spark across to the knuckle which is connected by the body to the earth.

15. *Why is it that a plate electrical machine will not work well in damp weather?*

See bottom of page 272.

16. *If you have a positively charged brass rod, and a piece of gilt paper fastened to the end of a dry glass rod, how would you charge the gilt paper with negative electricity?*

Using the dry glass rod as a handle, bring the gilt paper near the positively charged rod. Touch the paper with the finger, and remove the finger. The paper now has a negative charge. The paper, when near the rod, becomes charged by induction. The induced positive charge flows to earth through the finger and the body of the operator. The induced negative charge remains on the paper when the paper is removed from the rod.

17. *Three insulated metal balls, A, B, and C, are placed in a line, A and B in contact, C a little way off. C is positively electrified, and then A and B are separated. What are now the electrical states of A and B?*

A will be positively, and *B* negatively, electrified. While *A* and *B* are in contact, they form one conductor. *C* acts inductively on *A* and *B*, repelling positive electricity to *A* and attracting negative electricity to *B*. In this state *A* and *B* are separated, and their charges cannot reunite.

18. *Two pith balls hang side by side by two damp cotton threads. State and explain what happens when an electrified glass rod is brought gradually near the two pith balls.*

By induction the lower surfaces of the pith balls are charged negatively; the corresponding positive charges escape through the damp threads. The negatively charged pith balls repel each other, and therefore they will move away from each other.

19. *If an electrified piece of metal is made to touch a gold-leaf electroscope, the leaves separate; and on taking the metal away, the leaves remain separated. But if the electrified metal is only brought near to the electroscope, and then taken away, the leaves separate when the electrified metal is near, but fall together when it is taken away. Why is there a lasting effect on the gold leaves in one case, and only a temporary effect in the other case?*

In the first case, the disc of the electroscope, the conducting wire, and the gold leaves, are all charged by contact, and retain their charges after the metal is taken away; therefore the gold leaves remain separated. In the second case, the opposite charges induced by the approach of the metal reunite when the metal is removed; therefore the leaves fall together again.

20. *Two Leyden jars charged in the ordinary way are held one in each hand by the outer coatings. What takes place when the knob of one is made to touch the outer coating of the other, and what is the subsequent condition of each jar?*

The first jar will be discharged because its two coatings are brought in contact through the arms and body of the operator. The other jar will remain charged.

21. *An insulated conductor A is brought near to the cap of a gold-leaf electroscope charged positively. Explain what will happen* (1) *if A is unelectrified,* (2) *if A is charged positively,* (3) *if A is charged negatively.*

(1) The gold leaves will partially collapse. By induction *A* will receive a negative charge on the side nearest the electroscope, and a positive charge on the side remote from the electroscope. The negative charge on *A* will react on the electroscope so as to attract some of the positive electricity from the leaves ; therefore the leaves will partially collapse.

(2) The leaves will diverge more, because the positive electricity of *A* will repel some of the positive electricity of the electroscope to the leaves, thus causing them to diverge more.

(3) The divergence of the leaves will diminish. The negative charge on *A* will attract the positive charge on the electroscope thereby diminishing the quantity of positive electricity on the leaves ; therefore the leaves will partially collapse.

22. *If a rod of sealing wax is rubbed with a piece of flannel, the leaves of an electroscope diverge when either the wax or the flannel is separately brought near to it, but not if the wax and the flannel are presented to it together before being separated. What conclusions do you draw from these facts?*

If sealing wax is rubbed with flannel, they will each show electrification when they are brought separately to the disc of an electroscope. The wax will be negatively electrified, and the flannel positively electrified. But if they are brought *together* to the electroscope, the leaves will not diverge at all. Since the resultant effect is nothing, the inference is that positive and negative charges are present in equal quantities.

23. *A gold-leaf electroscope is placed upon an insulated brass plate. When the plate is electrified, will the leaves of the electroscope show any divergence? Give reasons for your answer.*

If the plate is *strongly* electrified, the leaves will diverge, because each

leaf receives by induction a charge opposite in kind to that on the plate, so that the charges on the leaves are alike in kind.

24. *Why is it impossible to charge a Leyden jar strongly unless its outer coating is connected with the earth?*

When the outer coating of a Leyden jar is connected to earth, the electricity induced on this coating by the charge on the inner coating, and similar in kind to the charge on the inner coating, is repelled to earth as fast as generated. Under these conditions, the induced charge, which is dissimilar to the charge on the inner coating, is able to hold bound a much greater quantity of electricity on the inner coating than it could hold if the outer coating were insulated. In short, by connecting the outer coating to earth, the *capacity* of the jar for electricity is greatly increased.

Laboratory Exercise. Page 293.

1. *Set up a Daniell cell. Allow the current to flow for one hour through a galvanometer, and observe the variations in the strength of the current every* 10 *minutes. Determine the change in weight of the plates by weighing them at the beginning and end of the hour.*

Metal plates with wires attached suitable for a quart Daniell cell, together with a glass jar and a porous pot, may be procured from manufacturers and dealers for about fifty cents. Assuming these materials on hand, the first thing to do is to amalgamate the zinc plate, unless this has already been done. For this purpose immerse the plate in mercury for a moment, and rub the mercury against the surface of the plate with a cloth. Lift the plate out of the mercury, taking care to brush from it every speck of loose mercury. The zinc plate is now ready for use.

Prepare a saturated solution of copper sulphate by dissolving 40 parts by weight of copper sulphate in 100 parts by weight of water. The sulphate should be pulverized in a mortar before it is added to the water, and then the water should be vigorously stirred, or put into a bottle and shaken till the sulphate is dissolved. Solution is promoted by gently warming the water.

Prepare some dilute sulphuric acid by pouring 1 part by weight of concentrated acid into 10 parts by weight of water. Pour the acid in slowly, and stir the mixture. *Do not pour water upon the acid.* Much heat is developed when the acid and the water come together; and if water is poured upon the acid, the water may be heated to boiling, and spatter the acid.

Place the porous pot (previously soaked in water for some time) into

the glass jar, fill the pot about half full with dilute acid, and the jar about two thirds full with the solution of copper sulphate.

Put the zinc plate into the porous pot, and the copper plate into the glass jar. The cell is complete, and a current will flow through the wires from copper to zinc as soon as they are connected.

The current increases in strength for a little time, and then remains very nearly constant for a long time. The zinc plate is slowly consumed, the zinc uniting with the acid and forming zinc sulphate. Copper in a state of fine subdivision is slowly deposited on the copper plate.

The source of this copper is the copper sulphate in solution. When the current begins to grow weaker, we may infer that a fresh supply of copper sulphate is needed.

The gain in weight of the copper plate is about equal to the loss in weight of the zinc plate for the same time.

When the current is no longer needed for use, the porous pot with its contents should be removed from the glass jar, the acid poured into a bottle for future use, and the zinc plate, after being washed with water, laid away for future use.

NOTE. — The Leclanché cell (Fig. 223), or the more expensive bichromate cell (Fig. 222), is preferable to the Daniell cell when only a brief use of a current is desired. The cells sold under the names of the "Samson" and the "Gonda" are of the Leclanché type. The cell called the "Grenet" is a bichromate cell. The teacher should be provided with one or more of these cells ready for immediate use.

Class-room Exercises. Page 293.

1. *Describe with a sketch a Daniell cell. What is the source of the energy of the current?*

See § 276. The source of the energy is the oxidation of the zinc plate. When this change occurs, the potential energy of chemical separation is converted into the energy of the current.

2. *Describe a Leclanché cell. Which cell would you use for ringing an electric bell, the Leclanché or the Daniell? Why?*

§ 276. The Leclanché cell is to be preferred to the Daniell for ringing an electric bell, because it is cheaper, needs no attention for months, and depolarizes rapidly after it has rung a bell.

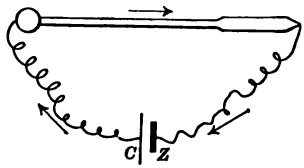
3. *What is meant by polarization? How is it prevented?*

See § 275.

4. *Why and how are zinc plates amalgamated?*

See § 274.

5. *How would you connect a poker by wires to the plates of a Daniell cell so as to make the current pass through the poker from the handle to the point? Give a sketch.*



Connect the copper plate to the handle of the poker, and the zinc plate to the point. See sketch; where C = copper plate, Z = zinc plate.

6. *If a charged battery is to be kept ready for use, why should the ends of the battery be kept disconnected outside the battery?*

If the ends of the battery are connected, the current will flow, and the materials of the battery will be gradually consumed. Therefore the ends should be kept disconnected except when the current is wanted for some useful purpose.

7. *What is meant by the terms open circuit and closed circuit?*

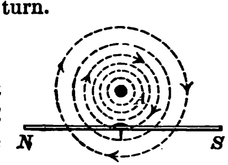
See § 271.

Laboratory Exercise. Page 303.

1. *Make a solenoid by winding insulated wire round a cylindrical lamp chimney, and study its magnetic properties by the method suggested in Fig.* 233, (1) *when the solenoid contains no iron core,* (2) *when it contains a soft iron core.*

A glass tube 8 or 10 in. long and about half an inch in bore is better than a lamp chimney for this experiment. The tube should be closely wound with No. 20 B. & S. gauge insulated copper wire. The soft iron core should be long enough to project a little at each end beyond the tube. Keys, nails, etc., may then be suspended from the ends. The magnetic properties of the solenoid may be made quite evident in various ways without taking the trouble of suspending the solenoid, as indicated in Fig. 233. A simpler mode of suspension is to bring the ends of the wire together at the middle of the solenoid, and twist them so as to form an axis about which the solenoid can turn.

Class-room Exercises. Page 303.



1. *A wire lies east and west directly over a compass needle. How is the needle affected when a strong current flows through the wire* (1) *from west to east,* (2) *from east to west?*

Let A be a cross-section of the wire, N S the compass needle; and suppose that the experiment is performed in a locality where the needle points due north (or a place of zero declination).

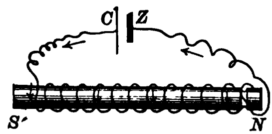
If the current flows through the wire from west to east (from the reader through the paper), then the lines of force around the wire will be as shown in the figure, and will simply tend to increase the horizontal strength of the earth's field. Therefore the needle will not move.

If the current is reversed, the lines of force around the wire will be reversed, and will act contrary to the earth's lines of force. If the current is strong, it will overcome the effect of the earth's field, and cause the needle to rotate through 180°.

If the experiment is performed in a locality where the needle has a declination (either easterly or westerly), the effect of a current from west to east will be to *diminish* by a certain amount the angle of declination, and the effect of a strong current from east to west will be to cause the needle to rotate through an angle somewhat less than 180°.

2. *If you have a voltaic cell, insulated wire, and a bar of soft iron with a notched end, how would you proceed to magnetize the iron so that the notched end should be a N. pole? Give a diagram.*

Wind the wire many times round the bar, and send a current from the cell through the wire so that the current, as viewed from the notched end, shall flow in the direction opposite to that of the hands of a watch.



3. *Describe a relay in telegraphy and its use.*

See § 286.

4. *Describe and sketch an electric bell which will ring as long as the current is closed.*

See § 285.

5. *Give a diagram of an astatic needle, and explain why the use of such a needle makes a galvanometer very sensitive.*

See § 280.

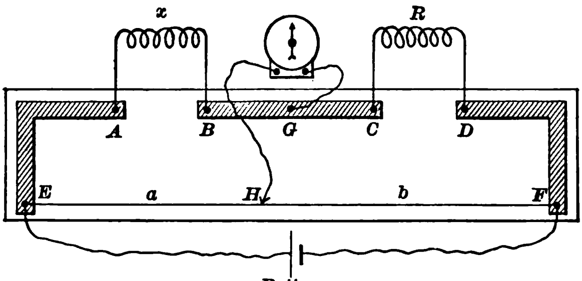
Laboratory Exercises. Page 316.

1. *Measure the resistance of a coil of wire by the method of Wheatstone's Bridge.*

If a resistance box is not provided, a form of Wheatstone's bridge known as the *slide-wire* bridge may be used. The sketch given on the next page will serve to make clear its mode of action.

Upon the upper surface of a hardwood base, flat strips of brass are fastened, as seen in the figure. Binding-posts are placed at A, B, C, D, E, F, and G. The coil of wire whose resistance x is to be found, is

inserted between A and B. A coil of wire whose resistance R in ohms is known, is inserted between C and D. The value of R should not differ



greatly from that of X. The battery wires are connected to the binding-posts E and F. One wire from the galvanometer is clamped at G. The end of the other wire from the galvanometer is fastened to a slider at H, which by pressure can be put in metallic contact with a German silver wire stretched between the binding-posts E and F.

The distances, a and b, from H to E and F respectively, are measured on a millimeter scale (not shown in the sketch). When all is ready, contact at H is made by pressing the slider. Usually a swing of the galvanometer needle will be observed. The position of H is then changed until on making contact no motion of the needle can be detected. The distances a and b are then measured, and the value of x is found from the proportion

$$x : \mathrm{R} = a : b.$$

A slide-wire bridge similar to that here described is furnished by the Zeigler Electric Company, Boston, Mass., for $4.25. To obtain accurate results a sensitive galvanometer is essential. The astatic galvanometer sold by the Zeigler Company for $3.50 is sufficiently sensitive for school purposes.

2. *Prove by experiment that the arrangement of cells in series gives the strongest current when the external resistance is large.*

Arrange the cells of the battery in series, and make the external resistance large by inserting in the circuit a galvanometer and a coil of fine wire. Note the deflection of the needle.

Join the cells in parallel circuit, and note the deflection. Join the cells partly in series and partly in parallel. It will be found that the strongest current is obtained when the cells are joined in series.

Class-room Exercises. Page 316.

1. *Find the resistance of* 8,000 *meters of iron wire, the diameter of which is* 0.2 *cm., specific resistance of iron* 0.1 *ohm.*

$R = \frac{kl}{s}$ (§ 290). Here $l = 8000$, $s = \pi r^2 = 100\,\pi = 314$ qmm., $k = 0.1$.
Therefore $R = \frac{800}{314} = 2.548$ ohms.

2. *Compare the resistances of two copper wires, one of which is* 100 *times as long as the other and has a cross-section one fourth as large as that of the other.*

The resistances are as 400 to 1.

3. *An iron wire is* 200 *meters long and* 3 *mm. in diameter. A copper wire is* 50 *meters long and* 1 *mm. in diameter. Compare their resistances. Specific resistance of iron* = 0.1, *of copper* = 0.016.

Let R = resistance of iron wire, R' = resistance of copper wire.

Then $$R = \frac{200 \times 0.1}{\pi\,(\frac{3}{2})^2},\quad R' = \frac{50 \times 0.016}{\pi\,(\frac{1}{2})^2},$$

Therefore $$\frac{R}{R'} = \frac{200 \times 0.1 \times 4}{9 \times 50 \times 0.016 \times 4} = \frac{0.4}{0.144} = \frac{25}{9}.$$

4. *What is the resistance of a column of mercury* 1 *meter long and* 1 *sq. mm. in cross-section? Specific resistance of mercury* = 0.943.

0.943 ohms.

5. *Two exactly equal pieces of copper are drawn into wire; one wire is* 10 *ft. long, the other* 20 *ft. long. If the resistance of the shorter wire is* 0.5 *ohm, what is the resistance of the longer wire?*

Since the two wires have equal volumes, the 20 ft. wire will have *half* the cross-section of the 10 ft. wire. The longer wire will have *twice* the resistance of the shorter wire because it is twice as long, and also *twice* the resistance of the shorter wire because its cross-section is half as great. Therefore resistance of the longer wire = $0.5 \times 4 = 2$ ohms.

6. *A piece of copper wire* 100 *ft. long weighs* 1 *lb. Another piece of copper wire* 500 *ft. long weighs* 4 *oz. What are the relative resistances of the two wires?*

If the wires had the same weight, the cross-section of the second wire would be $\frac{1}{5}$ as great as that of the first wire. But the second wire weighs only 4 oz. Therefore its cross-section is only $\frac{1}{20}$ as great as that of the first wire. Therefore for equal lengths the resistance of the second wire is 20 times as great as that of the first wire. Since it is 5 times as long, its resistance is 100 times as great as that of the first wire.

7. *The E. M. F. of a Daniell cell is* 1.02 *volts; the internal resistance is* 2 *ohms; the external resistance* 1 *ohm. What is the strength of the current?*

$$C = \frac{1.02}{1+2} = 0.34 \text{ ohms.}$$

8. *If* 100 *cells like that in Ex.* 7 *are joined in series, and the external resistance is* 200 *ohms, what is the strength of the current?*

$$C = \frac{102}{200 \times 200} = 0.255 \text{ ohms.}$$

9. *A battery of* 12 *Grove cells is arranged in* 3 *rows of* 4 *cells each. The* 4 *cells of each row are joined in series, and the cells of each row are joined to those of the other rows in surface. The E. M. F. of each cell is* 1.9 *volts, and its internal resistance is* 3 *ohms. The external resistance is* 6 *ohms. Find the strength of the current.*

$$C = \frac{4 \times 1.9}{6 + \frac{4 \times 3}{3}} = 0.76 \text{ ohms.}$$

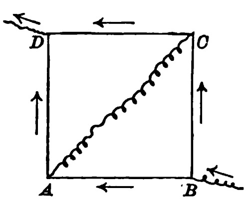
10. *Fifty cells of a battery in which the E. M. F. of each cell is* 1 *volt, and the internal resistance* 4 *ohms, are arranged in* 5 *rows of* 10 *each. Find the current when the external resistance is* (1) 12 *ohms, and* (2) 32 *ohms.*

$$(1)\ C = \frac{10}{12 + \frac{4 \times 10}{5}} = 0.5 \text{ ohms.} \quad (2)\ C = \frac{10}{32 + 8} = 0.25 \text{ ohms.}$$

11. *How would you arrange* 10 *equal cells, each having a resistance of* 2 *ohms, in order to obtain the strongest current through an external resistance of* 2 *ohms?*

The cells should be arranged 5 in series and 2 in surface, or 5 in surface and 2 in series. In either case $C = \frac{5\,E}{7}$.

12. *A wire whose resistance is* 4 *ohms is bent into the form of a square ABCD, the two ends being soldered together. Find the resistance of the system when the current enters at B and leaves at D. Will the resistance be altered if the corners A and C are connected by another wire?*



The system of wires $ABCD$ forms a divided circuit. Each of the branches BAD and BCD has a resistance of 2 ohms. Their joint resistance given by the formula (5) in § 291 is 1 ohm. This resistance will not be modified if A and C are connected by a wire, because

(by § 292) A and C are at the same potential, and no current will flow through the wire joining A and C.

13. *Two wires equal in length and thickness, one of iron and the other of platinum, are soldered together, and a current is sent through them. The potential difference between the ends of the entire wire is* 6 *volts. What is the potential difference between the free iron end and the soldered junction?*

Taking 0.10 and 0.11 as the specific resistances of iron and platinum respectively, it follows that the resistance of the iron is $\frac{10}{21}$ of the total resistance. Therefore (by § 292) the fall of potential in the iron will be $\frac{10}{21}$ of the total fall of potential (6 volts). $\frac{10}{21} \times 6 = 2.857$ volts.

14. *The external part of a circuit consists of two wires joined to the battery in multiple arc (see Fig.* 245*). Their resistances are* 10 *and* 16 *ohms respectively. The strength of the current in the battery is* 2 *amperes. Find the strength of the current in each wire.*

From the formulas at bottom of page 309,

$$C_1 = \frac{2 \times 16}{10 + 16} = 1.23 \text{ ohms.} \quad C_2 = \frac{2 \times 10}{10 + 16} = 0.846 \text{ ohms.}$$

15. *The total resistance of a circuit is* 18 *ohms. What change in the strength of the current will be produced if two points of the circuit between which the resistance is* 12 *ohms are joined by a wire of* 4 *ohms resistance?*

The divided circuit has a resistance of 3 ohms, so that the total resistance of the circuit is reduced from 18 ohms to 9 ohms. The strength of the current will be doubled.

16. *A galvanometer of* 90 *ohms resistance is shunted by a shunt of* 10 *ohms. If the potential difference between its terminals is* 45 *volts, find the resistance of the shunted galvanometer, and the strength of the current which flows through it.*

$$R = \frac{90 \times 10}{90 + 10} = 9 \text{ ohms.} \quad C = \frac{45}{9} = 5 \text{ amperes.}$$

17. *Twelve similar cells are arranged in series, and the poles of the battery are connected by a wire whose resistance is* 240 *ohms. E. M. F. of each cell* = 1 *volt; internal resistance* = 3 *ohms. By accident three of the cells are placed with their poles inverted. What is the strength of the current (see* § 288*)?*

$$C = \frac{9 - 3}{240 + 36} = 0.0217 \text{ ohms.}$$

18. *Six similar cells are arranged in series, and the circuit completed through a coil of wire and a galvanometer. The resistances of*

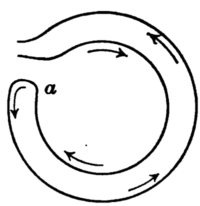
the battery, the coil, and the galvanometer are 10, 50, *and* 20 *ohms respectively. If the potential difference between the terminals of the galvanometer is* 2 *volts, find the E. M. F. of each cell.*

Since a resistance of 20 ohms corresponds to a fall in potential of 2 volts, and the total resistance in circuit is 80 ohms, the total E. M. F. is 8 volts. Since the six cells are arranged in series, the E. M. F. of each cell is equal to $\frac{8}{6}$, or 1.33 volts.

19. *An insulated wire is wound round a glass tube AB from end to end, and a current is sent through it so that to an observer looking at the end A the current flows round the wire clock-wise. A rod of soft iron is held* (1) *inside the tube,* (2) *outside but parallel to the tube. What in each case will be the magnetic pole of the end nearest the observer?*

(1) The end of the rod at A will be a south pole. (2) The end of the rod nearest A will be a north pole.

20. *An insulated wire is wound round a wooden cylinder AB from A to B. How would you wind it back from B to A* (1) *so as to increase,* (2) *so as to diminish the magnetic effects which it produces when a current is passed through it? Illustrate your answer by a diagram.*



(1) Continue to wind the wire so that the direction of the winding, as viewed from B, shall be the same as before.

(2) Reverse the direction of the winding from B to A so that the direction of the current shall be opposed to that of the first layer of wire. If the first layer of wire ends at a (see sketch), bend the wire on itself, so that the current shall flow in the direction of the outer arrows.

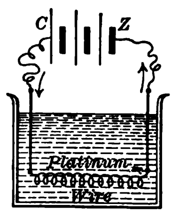
Class-room Exercises. Page 325.

1. *Why is it that if the poles of a battery are joined by a long, thin wire, the battery does not get so hot as when a short, thick wire is used?*

The great resistance of the long, thin wire causes the current to be much weaker than when a short, thick wire is used. Therefore the heating effect on the battery, being proportional to the square of the strength of the current, is much less.

2. *How could you boil water by means of a current from a battery? Give a sketch of the arrangement you would use.*

Join the battery in circuit with a coil of platinum wire. Choose the wire of such size and length that the current raises it to a red heat. Immerse the coil of wire in water. The heat generated will be transferred to the water, and raise its temperature finally to the boiling point, provided the quantity of water in the vessel is not too great.



3. *The wire which connects the plates of a Daniell cell gets hotter when the plates are brought nearer each other. Explain why.*

Because bringing the plates of the cell nearer together diminishes the internal resistance of the cell, and thus increases the strength of the current.

4. *The same current flows through a thick copper wire and a thin copper wire, joined end to end, and of such lengths that they have the same resistance. Which wire will become hotter? What is the reason?*

The thin copper wire will become hotter than the thick copper wire. Since the current in each wire has the same strength, and the resistances of the two wires are equal, the same quantity of heat will be generated in each wire. But the thick wire will weigh much more than the thin wire, both because it is thicker and because it is longer than the thin wire. For example, if the cross-section of the thick wire is 10 times that of the thin wire, the thick wire must be 10 times as long as the thin wire in order to have the same resistance; therefore it will weigh 100 times as much. Hence the same quantity of heat will raise its temperature much less.

It is true the thick wire will lose more heat by radiation than the thin wire, but this loss will not offset the effect of its greater weight.

5. *The resistance of two wires A and B made of the same metal are as 2 : 3. What are the relative quantities of heat developed in the wires* (1) *when fastened end to end so that the same current passes through both wires,* (2) *when arranged in multiple arc?*

(1) As 2 to 3. (2) As 3 to 2. In the second case, let C_1 denote the strength of the current in the wire A, C_2 the strength of the current in the wire B, C the total strength of the current, H_1 and H_2 the quantities of heat developed in the two wires respectively. Then

$$C_1 = \tfrac{3}{5}\,C\,,\; C_2 = \tfrac{2}{5}\,C\,,\; H_1 = (\tfrac{3}{5}\,C)^2 \times 2\,,\; H_2 = (\tfrac{2}{5}\,C)^2 \times 3\,;$$

Therefore $H_1 : H_2 = \tfrac{9}{25} \times 2 : \tfrac{4}{25} \times 3 = 18 : 12 = 3 : 2.$

6. *The poles of a cell are joined by two wires similar in all respects*

except that one is longer than the other. In which wire is the greater amount of heat produced, and why?

The greater amount of heat is produced in the longer wire because its resistance is greater.

7. *The E. M. F. of a battery is* 18 *volts, and its internal resistance is* 3 *ohms. The difference of potential between its poles when they are connected by a wire A is* 15 *volts, and falls to* 12 *volts when A is replaced by another wire B. Compare the quantities of heat developed in A and B in equal times.*

In the first case, 18 — 15, or 3 volts, is spent in forcing the current through the battery resistance of 3 ohms; in the second case, 18 — 12, or 6 volts, is so spent. Therefore, if C_1 and C_2 denote the current strengths in the two cases, $C_1 = 1$ ampere, $C_2 = 2$ amperes. Now the heating effect varies as C^2R, or since $E = CR$, as EC, where E denotes the potential difference corresponding to the resistance R.

In the first case, $E = 15$, $C = 1$.
In the second case, $E = 12$, $C = 2$.

$\therefore$ heat generated in wire A : heat generated in wire $B = 15 : 24$.

8. *A coil of wire whose resistance is* 35 *ohms is traversed by a current whose strength is* 0.4 *ampere. What amount of heat is generated in the wire each second?*

Amount of heat $= C^2R \times 0.24 = 0.16 \times 35 \times 0.24 = 1.344$ calories.

9. *How much heat is developed in* 1 *hour by a current of* 9 *amperes in a wire the potential difference of whose ends is* 300 *volts.*

Resistance of wire $= \frac{300}{9} = \frac{100}{3}$ ohms. From $H = C^2Rt \times 0.24$, we have $H = 81 \times \frac{100}{3} \times 3600 \times 0.24 = 2{,}332{,}800$ calores.

10. *What horse power is required to maintain through a resistance of* 373 *ohms* (1) *a current of* 5 *amperes,* (2) *a current of* 10 *amperes?*

(1) No. of watts consumed $= C^2R = 25 \times 373$;

H. P. required $= \dfrac{25 \times 373}{746} = 12\frac{1}{2}$.

(2) No. of watts consumed $= 100 \times 373$;

H. P. required $= \dfrac{100 \times 373}{746} = 50$.

11. *What is the equivalent in horse powers of a current of* 1000 *amperes flowing under a pressure of* 1 *volt?*

Energy consumed per second $= C^2R = EC = 1000$ watts.

H. P. $= 1.34$.

12. *The strength of the current flowing through a coil of wire is* 30 *amperes, and the potential difference between the ends of the coil is* 60

volts. How much energy is consumed per hour in joules, and also in foot-pounds? How much heat is developed?

Energy consumed per hour $= ECt = 30 \times 60 \times 3600 = 6{,}480{,}000$ joules $= 6{,}480{,}000 \times 0.727 = 4{,}710{,}960$ ft.-lb.

Heat developed $= 6{,}480{,}000 \times 0.24 = 1{,}547{,}200$ calories.

13. *How many watts must be expended to send a current of* 10 *amperes through a resistance of* 15 *ohms?*

$$C^2R = 100 \times 15 = 1500 \text{ watts.}$$

14. *A current of* 2 *amperes is allowed to flow for* 20 *minutes through a coil of wire of* 8 *ohms resistance, immersed in* 300 *grams of water. What is the rise in the temperature of the water?*

$$\text{Heat generated} = 4 \times 8 \times 20 \times 60 \times 0.24 \text{ calories.}$$

$$\text{Rise in temperature} = \frac{4 \times 8 \times 20 \times 60 \times 0.24}{300} = 30^\circ.72.$$

15. *In order to determine the strength of a current, it is made to pass through a coil of wire of* 40 *ohms resistance, placed in a calorimeter containing* 200 *grams of water. After* 10 *minutes the temperature of the water had risen* 15°. *Find the strength of the current.*

Amount of heat received by the water $= 200 \times 15$ calories.

Let $x =$ the strength of the current; then

$$40 \times 600 \times 0.24x^2 = 200 \times 15, \text{ whence } x = 0.72 \text{ ampere.}$$

16. *The resistance of a conductor is halved and the current is doubled. How is the heat generated in the conductor affected?*

The heat generated is doubled.

17. *If the current in a conductor is quadrupled, how must the resistance of the conductor be altered in order that the amount of heat generated may remain unchanged?*

If the current is quadrupled, the quantity of heat generated is made 16 times as great as before. To offset this increase, the resistance must be made $\frac{1}{16}$ as great as before.

18. *An arc lamp takes a current of* 10 *amperes, and its voltage is* 50 *volts. What power does it absorb?*

$$C^2R = EC = 50 \times 10 = 500 \text{ watts.}$$

19. *A glow lamp of* 16*-candle power takes a current of* 0.75 *ampere, and its voltage is* 100 *volts. Find the number of watts per candle power absorbed by the lamp and the amount of heat generated in* 1 *hour.*

The lamp absorbs $\frac{100 \times 0.75}{16}$ or 4.7 watts per candle power, very nearly. Heat generated per hour $= 64{,}800$ calories.

20. *If a current of* 0.75 *ampere flows through an Edison lamp when* 108 *volts are maintained at its terminals, how many foot-pounds of work per minute are expended on the lamp? How many such lamps can be made to glow with the expenditure of* 5 *horse power?*

$0.75 \times 108 \times 60 \times 0.727 = 3533$ ft.-lb., very nearly.

5 H. P. $= 5 \times 33000$ ft.-lb. of work per minute.

No. of lamps maintained by 5 H. P. $= \frac{165000}{3533} = 46$.

21. *A* 16*-candle-power glow lamp takes a current of* 1 *ampere with an E.M.F. of* 52 *volts. Find the number of watts absorbed per candle, and the H.P. required to supply* 200 *of these lamps in parallel circuit.*

No. of watts per candle-power $= \frac{52}{16} = 3.25$.

Power required to run 200 lamps $= 10{,}400$ watts $= 14$ H. P., very nearly.

22. *Calculate the power required to light* 80 *glow lamps, the voltage of each being* 65 *volts, and the strength of current required being* 0.8 *ampere.*

$$\text{Power required} = \frac{0.8 \times 65 \times 80}{746} = 5.6 \text{ H. P., nearly.}$$

23. *A conductor carrying a current divides into two branches whose resistances are in the ratio of* 4 : 5. *Compare the amounts of heat generated in the branches (see* § 291).

The strengths of the currents in the two branches are as 5 to 4. The amounts of heat generated are as $5^2 \times 4$ to $4^2 \times 5$, or as 5 to 4.

24. *A battery of* 10 *cells, the resistance of each of which is* 4 *ohms, is arranged in series. What must be the resistance of a wire in order that the current from the battery when sent through the wire may expend nine tenths of its energy in heating the wire?*

Since the heating effect varies as the value of C^2R, and C has the same value throughout the circuit, it is evident that the resistance of the wire should be nine tenths that of the battery, or 36 ohms.

25. *The internal resistance of a battery of* 12 *cells joined in series is* 36 *ohms. The E. M. F. of each cell is* 2 *volts. The strength of the current when the poles of the battery are joined by a certain wire is* 0.4 *ampere. Find the resistance of the wire, and the number of calories of heat set free in the wire in one hour.*

Let $x =$ the resistance of the wire;

Then by Ohm's Law $0.4 = \frac{24}{36 + x}$, whence $x = 2.4$ ohms.

$$\text{Heat developed per hour} = (0.4)^2 \times 2.4 \times 3600 \times 0.24$$
$$= 331.776 \text{ calories.}$$

Laboratory Exercises. Page 333.

1. *Cover a silver coin with a coating of copper. After the coin is well coated with copper, reverse the current. Describe and explain what then takes place.*

Fasten to the silver coin a copper wire. Dip the coin into a solution of potash to remove any grease that may be adhering to it. Wash the coin with water, and dip it into sulphuric acid to remove any oxide that may have formed on its surface. Again wash the coin in water, taking care not to touch its surface with the fingers. Put a strong solution of copper sulphate into a glass tumbler, and immerse the coin in the solution. Join the wire attached to the coin to the zinc plate of a Daniell cell. Attach the wire leading from the copper plate of the cell to another plate of copper, and dip this plate into the solution. The coin will receive a coating of copper. On reversing the current which passes through the solution, the copper will gradually disappear from the coin.

2. *Make a copperplate copy of a medal or similar object.*

Warm a large lump of beeswax very slowly till the whole mass has become quite soft. Press the medal to be copied into the soft wax, place them upon a table, the medal uppermost, and place upon the medal a heavy weight. Wait an hour or more, till the wax has become cold throughout its mass. Separate the medal from the wax. A perfect but reversed copy of the medal will be seen in the wax.

This reversed copy of the medal is called the *mould.*

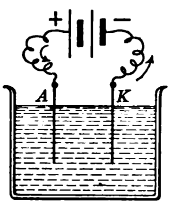
Brush over the surface of the mould powdered graphite (plumbago, black-lead), using for this purpose a camel's hair brush with a large, thick body of short hairs. Breathe upon the face of the mould occasionally to make the graphite adhere better, and blow away the superfluous graphite. The object of this operation is to render every part of the surface of the mould on which copper is to be deposited a good conductor of electricity.

Put into a vessel of suitable size a strong solution of copper sulphate. Join a copper wire to the mould so that the end of the wire shall not deface any part of the copy to be made, and yet be in metallic connection with the film of graphite. Place the mould in the solution of copper sulphate, and connect the wire which leads from it to the zinc plate of a battery. Place a plate of copper in the solution so as to face the impression in the mould, and connect it by a wire with the copper plate of the battery. Copper will now begin to be deposited over the black surface of the mould, and in time a layer $\frac{1}{16}$ of an inch thick will be formed. The time required depends on various circumstances, and

may be several days. When the layer is thick enough to retain its form perfectly, it should be separated from the mould, and greater stiffness given to it by the process of "backing." As backing material solder or type metal may be used. Press the copper electro face downwards into soft moist clay; moisten the upper surface with a solution of zinc chloride to make the solder adhere, then pour the melted solder upon the upper surface till you have a layer about $\frac{1}{4}$ of an inch thick. When the solder is cold, the operation of making a copy of a medal is complete.

Class-room Exercises. Page 333.

1. *Sketch and describe an arrangement by means of which a piece of platinum can be coated with copper.*



Place a solution of copper sulphate in a glass vessel. In this solution suspend two pieces of platinum. Connect one of them, A, with the + pole of a battery, and the other, K, with the − pole. The kathode K will soon become coated with pure copper.

2. *Explain with a diagram how you would proceed in order to plate an article with silver.*

See § 312.

3. *A battery is hidden from view, but the ends of wires connected with the poles of the battery are in sight. How can you ascertain which wire is connected with zinc and which with the copper pole of the battery by observing what takes place when the ends of the wires are dipped into a solution of copper sulphate?*

The wire which becomes coated with a deposit of pure copper is the kathode, and is connected with the zinc plate of the battery.

See §§ 306 and 307.

4. *How much copper will be deposited by a current of 3 amperes in 1 hour? Electro-chemical equivalent of copper, 0.000327 gram.*

$$W = Ctz = 3 \times 3600 \times 0.000327 = 3.5316 \text{ grams.}$$

5. *How long will it take a current of 1 ampere to deposit 10 grams of silver? Electro-chemical equivalent of silver, 0.001118 gram.*

$$t = \frac{W}{Cz} = \frac{10}{0.001118} = 8945 \text{ sec.} = 2^h\ 12^m\ 25^s.$$

6. *What is the strength of a current that will deposit 1 gram of copper in 2 hours?*

$$C = \frac{W}{tz}. \quad \text{Here } W = 1,\ t = 7200,\ z = 0.000327.$$

Therefore $C = 0.425$ ampere, very nearly.

7. *A battery of* 8 *cells joined in series is used to decompose water. How much zinc is consumed by the battery while* 1 *gram of hydrogen is set free? Chemical equivalents: hydrogen* 1, *zinc* 32.5.

Faraday's laws of electrolysis apply to the portion of the circuit within the battery as well as to the portion outside the battery. The amount of zinc consumed in each one of the 8 cells, while 1 gram of hydrogen is set free on the voltameter, is the same, and equal to 32.5 grams.

Therefore amount of zinc consumed $= 32.5 \times 8 = 260$ grams.

If 4 cells had been used instead of 8, only half as much zinc would have been consumed in setting free the gram of hydrogen; but the current would have been much weaker, and the operation would have taken a longer time.

"The electrolytic cell," as Professor S. P. Thompson says, "is the *converse* of the voltaic cell." Suppose a current of strength C flows through a cell of electro-motive force E. If C and E have the same direction (as in Fig. 1 below), an amount of energy equal to CE is added to the circuit. But if C and E have opposite directions (as in Fig. 2), an amount of energy equal to CE is withdrawn from the circuit, and expended in doing chemical work in the cell. Case 1 is that of the ordinary voltaic cell joined in series; case 2 is that of an electrolytic cell.

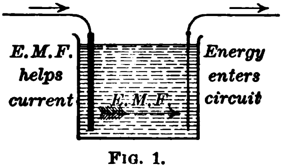


FIG. 1.

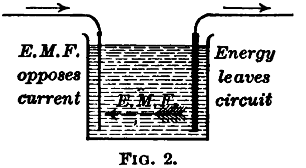


FIG. 2.

When a current is sent backwards through a Daniell cell which has been in action for some time, the action is reversed, copper is dissolved, and zinc is deposited on the zinc plate.

8. *A copper voltameter and a silver voltameter are included in the same circuit. After a certain time it is found that* 1 *gram of copper is deposited. How much silver will be deposited during the same time?*

3.42 grams.

9. *A current divides into two branches, each of which is carried through a solution of copper sulphate. If all the circumstances are the same except that the electrodes are in one case copper and in the other case platinum, will the two currents through the solutions be equally strong? Give reasons for your answer.*

The current through the branch with platinum electrodes will be weaker than the current through the branch with copper electrodes. For by electrolytic action the platinum kathode will soon be coated with copper, and a counter E. M. F. is produced which weakens the current flowing through this branch. In the other branch (with copper plates) there is no counter E. M. F., but merely the ordinary resistance of the liquid in the cell.

10. *A current is passed through a coil of wire and through a voltameter arranged in series with it. If the strength of the current is altered so that the heat produced per minute in the coil of wire is doubled, what change will be produced in the rate at which chemical action takes place in the voltameter?*

Heat produced varies as the *square* of the current; therefore, to double the quantity of heat, the current must be multiplied by $\sqrt{2}$. By Faraday's law the chemical action in the voltameter will also be multiplied by $\sqrt{2}$.

Review Exercises on Chapter VII.

1. *Define the terms declination and dip, as applied to a magnetic needle. What are isogonic lines?*

See § 242.

2. *Explain the use of a mariner's compass.*

See § 242.

3. *Define a magnetic pole of unit strength.*

See § 244.

4. *Describe an experiment which illustrates magnetic induction.*

See § 245.

5. *What is the difference in the behavior of soft iron and steel as regards magnetism?*

See § 246.

6. *How would you impart magnetism to a bar of steel?*

See § 247.

7. *State briefly the theory of molecular magnets. What are some of the facts that support it?*

See § 248.

8. *Define a line of magnetic force. Describe (by a sketch or otherwise) the general arrangement of lines of force on the field of a bar magnet.*

See § 250.

9. *Describe an experiment which leads us to divide bodies into conductors and non-conductors of electricity.*

See § 254.

10. *How can a body be charged by induction?*

See § 257.

11. *Describe the electrophorus and its action.*

See § 258.

12. *How would you use a gold-leaf electroscope for the purpose of ascertaining what kind of a charge a body has?*

See § 261.

13. *Explain the action of a plate electric machine.*

See § 262.

14. *Define the electrostatic unit of electric quantity.*

See § 263.

15. *How would you prove that electricity is wholly confined to the outer surface of a charged insulated body?*

See § 264.

16. *Describe an electric condenser and its action.*

See § 266.

17. *How did Franklin prove that the charges of a Leyden jar reside on the surface of the glass, not on the tinfoil coatings?*

See § 267.

18. *What is meant by saying that two electrified bodies have different potentials? Point out the analogy between difference of potentials and difference of water levels or of temperatures.*

See § 270.

19. *What is the source of the energy of an electric current generated by a voltaic cell?*

See § 272.

20. *What is polarization in a voltaic cell, its cause, and its remedy?*

See § 275.

21. *Describe a Daniell cell and its action.*

See § 276.

22. *What was Oersted's discovery?*

See § 278.

23. *What two classes of galvanometers are there? Describe some one galvanometer, mentioning its merits and defects.*

See § 280.

24. *Describe the magnetic field which exists around an electric current.*

See § 281.

25. ***What is a solenoid, and how does it behave?***

See § 282.

26. ***What is the effect of putting an iron core into a solenoid?***

See § 284.

27. ***Why are electromagnets usually made in the shape of a horseshoe and provided with an armature?***

See § 284.

28. *State Ohm's law. Apply it to two cells the zincs of which are joined by one wire and the coppers by another wire.*

See §§ 287 and 288.

29. *Define the C. G. S. units of current strength, quantity, electromotive force, and resistance.*

See § 289.

30. *Define the ampere, coulomb, volt, and ohm.*

See § 289.

31. *Prove that, when a current is shunted, the partial currents vary inversely as the resistances through which they pass.*

See § 291.

32. *Prove that the fall of potential along a conductor is directly proportional to the resistance passed over.*

See § 292.

33. *Describe a method of measuring the resistance of a wire.*

See §§ 293 and 294.

34. *When should voltaic cells be joined in series?*

When the external resistance is very large compared with the internal resistance.

35. *State Joule's law.*

See § 299.

36. *Define the watt, and show that the heat equivalent of* 1 *watt is* 0.24 *calories of heat per second.*

See § 300.

37. *What is a thermopile, and what is it used for?*

See § 305.

38. *Define the terms electrolysis, anode, kathode.*

See § 306.

39. *Describe the electrolysis of copper sulphate* (1) *if a platinum anode is used,* (2) *if a copper anode is used.*

See § 307.

40. *Explain why one Daniell cell is unable to decompose water, how-*

ever large it may be, while two cells joined in series will decompose water, however small they may be.

See § 309.

41. *Give an example of a secondary current. Describe the Planté storage cell.*

See § 310.

42. *State Faraday's laws of electrolysis.*

See § 311.

43. *Describe the process of electrotyping.*

See § 312.

44. *How are induced currents related to lines of force?*

See § 314.

45. *What are eddy currents?*

See § 316.

46. *Describe a commutator.*

See § 318.

47. *What are the essential parts of a dynamo?*

See § 317.

48. *The electric efficiency of a certain dynamo is 95%, and its commercial efficiency is 90%. Explain the meaning of these statements.*

See § 323.

49. *What is meant by saying that a dynamo is a reversible machine?*

See § 324.

50. *Why does the economic transmission of power by electricity require a high voltage?*

See § 326.

51. *Describe an induction coil and its action.*

See § 329.

52. *State and illustrate the principle of transformation.*

See § 330.

53. *Describe briefly the telephone.*

See § 332.

CHAPTER VIII.

SOUND AND LIGHT.

Class-room Exercises. Page 374.

1. *The report of a cannon is heard* 12 *seconds after the flash is seen. How far is the observer from the cannon? Temperature* 20° *C.*

20 Centigrade degrees = 36 Fahrenheit degrees.
∴ Vel. of sound at 20° C. = 1090 + 36 = 1126 ft. per second.
Distance of cannon = 1126 × 12 = 13,512 ft.

2. *The report of a cannon is heard* 10 *seconds after the flash is seen. The temperature of the air is* 62° *F. The distance of the cannon from the observer is* 11,500 *feet. Find the velocity of sound.*

Vel. of sound at 62° F. = 1150 ft. per second.

3. *A string makes* 256 *vibrations per second. How many vibrations would it make if its length were doubled? If its tension were doubled?*

128. $256 \times \sqrt{2} = 362$, very nearly.

4. *Find the wave lengths of the fundamental tone of an open organ pipe* 16 *feet long, and of its first two overtones.*

32 ft., 16 ft., $10\frac{2}{3}$ ft.

5. *A man standing before a precipice shouts, and hears the echo in* $4\frac{1}{2}$ *seconds. How far away is the precipice?*

The distance of the precipice is equal to the distance through which sound will travel in $2\frac{1}{4}$ seconds, or (at 0° C.) 2,353 feet.

6. *What effect on the pitch of an organ pipe will be produced by filling it with hydrogen instead of air?*

The fundamental tone emitted by the pipe will be about two octaves higher than that emitted when the pipe is full of air. In the formula $v = ln$, l is constant and equal to 4 times the length of the pipe (if a stopped pipe). Now v for hydrogen gas is about 4 times as great as for air (§ 339). Therefore n is about 4 times as great as in the case of air.

7. *A glass jar containing water responds most loudly to a tuning fork when the length of the column of air is* 17.5 *cm. The temperature at the time is* 20° *C. What is the frequency of the fork?*

Wave length $= 4 \times 17.5 = 70$ cm. $v = 33{,}200 + 20 \times 55$
$= 34{,}300$ cm.

Therefore $n = \frac{34300}{70} = 490.$

8. *Compute the velocity of sound in air if a column of air* 33.5 *cm. long reënforces most strongly a tuning fork of frequency* 256.

$$v = ln = 4 \times 33.5 \times 256 = 34304 \text{ cm. per sec.}$$

9. *Two sound waves, of the same amplitude, meet in the open air. What will happen if they meet in like phases? In opposite phases?*

If the two sound waves meet in like phases, they will combine to produce a resultant wave having twice the amplitude of either wave. If they meet in opposite phases, they will destroy each other, and the air at the place of meeting will not be disturbed. This is the physical meaning of the statement that "sound added to sound may produce silence."

10. *A sonometer string stretched by a* 16-*lb. weight gives a certain note. What weight will give a note an octave lower?*

A weight of 4 lb.

11. *Compare the fundamental tones emitted by a stopped organ pipe* 4 *feet long and an open organ pipe* 16 *feet long.*

The tone of the open pipe is an octave lower than that of the stopped pipe.

12. *Is the fundamental tone of an organ pipe the same in summer as in winter? Give reasons for your answer.*

The tone is higher in summer than in winter because the frequency of the tone varies directly as the velocity of sound in air; and a rise in temperature increases the velocity of sound.

13. *A certain wire* 1 *meter long, weighing* 8 *grams, and stretched by a weight of* 10 *kg., makes* 64 *vibrations per second. What is the frequency of a wire of the same material* 50 *cm. long, weighing* 16 *grams, and stretched by a weight of* 20 *kg.?*

The change in length from 1 meter to 50 cm. will increase the number of vibrations from 64 to 128. The change in weight from 8 grams to 16 grams will reduce the number of vibrations from 128 to $128 \times \sqrt{\frac{1}{2}}$. The change in the tension of the wire from 10 kg. to 20 kg. will increase the number of vibrations from $128 \times \sqrt{\frac{1}{2}}$ to $128 \times \sqrt{\frac{1}{2}} \times \sqrt{2}$, or 128. *Ans.*

14. *Why does the sound of a circular saw fall in pitch as the saw enters a log of wood?*

Because as the saw enters the log its rate of motion diminishes, and

therefore the frequency of the waves which it excites in the air is diminished.

15. *How would you make a sonometer wire vibrate so as to yield upper partial tones? Why are piano wires struck near the end rather than at the middle?*

See § 348. Piano wires are struck near the end in order that several overtones may be produced; these by combination with the prime tone give a richer quality to the music.

16. *If you have a string of catgut and wish to fit a sonometer with such lengths that they will under equal tension emit the successive tones of the diatonic scale, what lengths must be taken? Take for the lowest tone a length of* 96 *cm.*

$96, 85\frac{1}{3}, 76\frac{4}{5}, 72, 64, 57\frac{3}{5}, 51\frac{1}{5}, 48.$

Laboratory Exercise. Page 382.

1. *Measure the candle power of some source of light by means of a Bunsen photometer.*

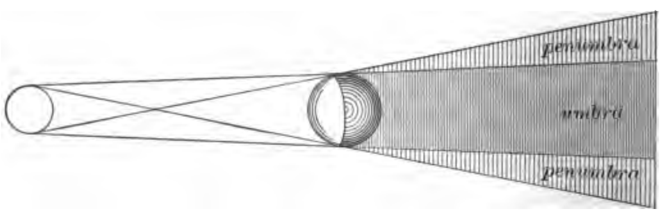
See § 367.

Class-room Exercises. Page 382.

1. *Under what conditions will the image of an object made by a pin-hole camera be equal in size to the object? Larger than the object?*

The image will have the same size as the object, if it is at the same distance from the pinhole as the object. The image will be larger than the object, if it is at a greater distance from the pinhole than the object.

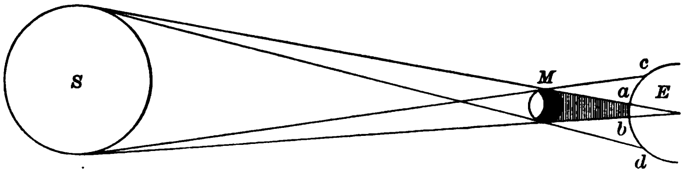
2. *Draw a diagram to illustrate the formation of a shadow by an opaque body which is larger than the luminous body.*



3. *Under what conditions will a spectator on the earth see an eclipse of the sun. Illustrate by a diagram.*

A spectator on the earth will see an eclipse of the sun when the moon comes between his eye and the sun. The eclipse will be total or partial according as it covers the whole of the sun's disc or only a part of it.

If the spectator is in the moon's umbra, or between a and b, the eclipse



will be total; if the spectator is in the moon's penumbra, or between a and c, or b and d, the eclipse will be partial.

4. *In one of Fizeau's experiments the distance of the mirror was* 8,663 *meters, the number of teeth in the wheel* 720, *and* 12.6 *revolutions per second caused the image to disappear. Find the velocity of light.*

The width of the teeth being equal to the width of the spaces, the time required for the wheel to turn through the width of a tooth was $\frac{1}{2} \times \frac{1}{720} \times \frac{1}{12.6}$ of a second, or $\frac{1}{18144}$ of a second. During this time the light traveled a distance of 2×8663, or 17,326, meters. Therefore

$$\text{Velocity of light} = 17326 \times 18144 = 314{,}262{,}944 \text{ meters.}$$

More recently Cornu has repeated Fizeau's experiment with more perfect apparatus, and obtained as the mean result,

300,340,000 meters, or about 186,600 miles per sec.

5. *Two lights, A and B, are* 40 *inches apart. The power of A is to that of B as* 9 : 16. *At what point between them must a screen be placed in order to be equally illuminated on both sides?*

$$P : P' = d^2 : d'^2. \quad \text{Also } P : P' = 9 : 16.$$

Therefore $d^2 : d'^2 = 9 : 16$, and $d : d' = 3 : 4$.

If d denote the distance of the screen from A, then $d' = 40 - d$, and we have $d : 40 - d = 3 : 4$, whence $d = 17\frac{1}{7}$ in.

6. *In measuring the illuminating power of a glow lamp by Bunsen's photometer the distances from the paraffine spot to the glow lamp and the standard candle were* 60 *cm. and* 15 *cm., respectively. What is the candle power of the lamp?*

The illuminating power varies as the square of the distance from the paraffine spot. Since the glow lamp is 4 times as far away as the candle, its power must be 16 times as great.

7. *A gas jet, when burning* 5 *cubic feet of gas per hour, placed* 50 *inches from a screen illuminates it equally with a candle placed* 15 *inches from the screen and burning at the rate of* 120 *grains per hour.*

The gas costs \$1 per 1,000 cubic feet and the candles \$0.25 per lb. Compare the cost of lighting a room with gas and candles, respectively.

If P and P' are the illuminating powers of the gas jet and the candle respectively, $P : P' = 2500 : 225 = 100 : 9,$

whence $P = \frac{100\ P'}{9}$, or equal to $11\frac{1}{9}$ candles.

The cost of one gas jet per hour is 0.5 of a cent.

$\frac{100}{9}$ candles consume per hour $\frac{100}{9} \times 120$, or $\frac{4000}{3}$ grains.

7,000 grains make 1 lb., and cost 25 cents.

Therefore the cost of $\frac{4000}{3}$ grains will be

$$\frac{4000 \times 25}{3 \times 7000} = 4.76 \text{ cents.}$$

Therefore the cost of lighting a room by gas is only about $\frac{1}{9}$ as much as when candles are used.

Laboratory Exercises. Page 392.

1. *Verify the mirror formula by experiments with a concave mirror.*

From the mirror formula, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

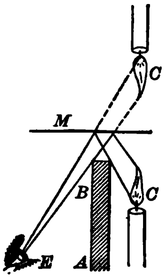
we obtain $v = \frac{uf}{u - f}$.

To verify this formula, we first measure the value of f, then assume a value for u; compute the corresponding value of v, and then see if a distinct image of an object placed at the distance u from the mirror is obtained at a distance from the mirror equal to the computed value of v.

To find the value of f, fit a cork as a slider to a meter rod, and mount the mirror upon the cork. Point the meter rod to the sun with the mirror facing the sun, and hold a small piece of cardboard in front of the mirror so that you obtain on the cardboard as bright and distinct an image of the sun as possible. Note the distance of the cardboard from the mirror. This distance is the value of f for the mirror, and is equal to half the radius of the mirror. It is best to use a mirror for which f is not more than 20 cm.

Fit two more corks as sliders upon the meter rod. Mount upon one of these corks a candle, and upon the other a card, taking care that the middle points of the card, the candle, and the mirror shall be in a straight line parallel to the meter rod. Clamp the rod in a horizontal position to a table. Place the mirror at one end of the rod. Place the candle at a distance from the mirror greater than its focal length, but

less than its radius. Place the card at a distance from the mirror considerably greater than its radius. Light the candle. Darken the room. Shift the position of the card or the candle till you obtain a distinct image of the candle flame on the card. Admit light to the room. Measure the distance u from the candle to the mirror, and the distance v from the card to the mirror.



Compute the value of v by substituting in the formula

$$v = \frac{uf}{u - f}$$

the values of u and f.

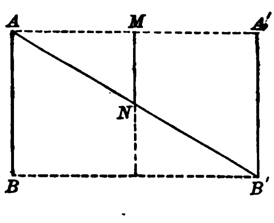
If sufficient care is taken, the computed and the observed values of v will nearly agree.

Class-room Exercises. Page 392.

1. *Show by a figure how you would place a plane mirror so as to see a candle flame placed behind an opaque object.*

In the figure AB is the opaque object, C the candle flame, M the mirror, E the eye, C' the image of the flame.

2. *A man 6 feet high sees his image in a plane mirror hung on a wall. The top of the mirror is 6 feet from the floor. Find its least length to enable the man to see his whole image in it.*



In the figure, AB is the position of the man, $A'B'$ that of his image, MN is the mirror. Assuming the eye of the man to be at A, the height MN of the mirror must be such that the field of view at the position of the image shall have a height of 6 ft. Since $AM = \frac{1}{2} AA'$, MN must be half of $A'B'$, or 3 ft.

3. *Two parallel mirrors are 2 feet apart, and a luminous point is midway between them. Show by a diagram on a scale of $\frac{1}{12}$ the position of the first three images formed in each mirror.*

The diagram must be drawn as illustrated in Fig. 307 ; the distance between the mirrors being made equal to $\frac{1}{12}$ of 2 ft., or 2 inches.

4. *A plane mirror is inclined to the floor of a room at an angle of 45°. Prove that the image of a man standing before it is horizontal.*

It is evident from Fig. 303 that the image of a straight line placed before a plane mirror makes the same angle with the plane of the mirror that the straight line itself makes ; and that the angle between the

straight line and its image is just twice as great as the angle between the straight line and the mirror. In the case supposed, the angle between the man and the plane of the mirror is 45°. Therefore the angle formed by the man and his image is 90°. That is, the image is horizontal.

5. *An object is placed* 15 *inches from a concave mirror whose radius is* 12 *inches. Find the position and nature of the image.*

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}. \quad \text{Here } u=15, f=6.$$

Whence $v = 10$ inches. The image is real and 10 in. from the mirror.

6. *An arrow* 4 *in. long is placed* 4 *in. from a concave mirror whose radius is* 12 *in. Find the position, nature, and magnitude of the image.*

In this case $u = 4, f = 6$. Whence $v = -12$. The image is virtual, is apparently 12 inches behind the mirror, and its length is 12 inches.

7. *The image of a candle flame placed* 10 *inches from a concave mirror is most distinctly outlined on a screen* 30 *inches from the mirror. Find the focal length of the mirror.*

In this case we have $\frac{1}{10}+\frac{1}{30}=\frac{1}{f}$, whence $f = 7\frac{1}{2}$ inches.

8. *Trace the changes in the position and nature of the image of a luminous point, as the point is moved from a great distance up to a concave mirror along its axis.*

First suppose that the distance of the luminous point from the mirror is so great that rays which strike the mirror are sensibly parallel to one another; then an image of the point is formed at the principal focus, or half way between the mirror and its center.

As the point moves towards the mirror, its image moves in the opposite direction. The point and its image meet and coincide at the center of curvature of the mirror. As the point moves from the center to the principal focus, the image moves from the center to an infinite distance. When the point is moved nearer to the mirror the image becomes virtual, and is at first situated apparently at a great distance behind the mirror. While the point is moved from the principal focus up to the mirror, the image moves also up to the mirror.

Laboratory Exercises. Page 401.

1. *Allow a beam of sunlight to fall on a glass prism so as to have total reflection from one of its faces. Then draw carefully a diagram representing the path of the rays through the prism.*

It is perhaps better to use a candle flame instead of the sun, as the

object. Take a prism with the angles each equal to 60°, and hold it a few inches below the level of the flame. Turn the prism till a good image of the flame is seen. The lower face of the prism is then nearly horizontal, and the face through which the rays of the candle enter the prism is nearly perpendicular to the direction of the rays. The image is formed exactly as in a common plane mirror, and the course of the rays may be represented readily by means of a diagram.

2. *Find the focal length of a convex lens* (1) *by throwing an image of the sun or some distant object on a screen,* (2) *by the "pin" method.*

(1) Fit two corks as sliders on a meter rod. On one of the corks mount the convex lens, and on the other a card. Point the meter rod towards the sun, and adjust the lens and the card so that a bright image of the sun appears on the card, and the heat is so great that the paper smokes. The distance from the lens to the card is the focal length of the lens.

(2) In the "pin" method replace the card by a pin. Point the meter rod towards some object several hundred feet away, such as a chimney. Rest one end of the meter rod on a firm support, hold the eye near the other end of the rod, and place the pin about as far from the eye as you are accustomed to hold a book when reading. On looking through the lens you will see an inverted image of the chimney. Move the eye from side to side; probably the pin and the image of the chimney will move relatively to each other. In that case change the position of the lens till there is no relative motion of the pin and the image. The distance from the lens to the pin when no relative motion occurs will be equal to the focal length of the lens.

In moving the lens so as to secure no relative motion of the pin and the image, it should be remembered that the more distant of two objects moves with respect to the nearer object in the same direction as that in which the eye is moved.

3. *Verify the formula given for a convex lens in* § 389.

The value of v from the lens formula (§ 389) is

$$v = \frac{uf}{u - f}.$$

To verify the formula, take a convex lens, obtain the value of its focal length f, and mount it on a cork slider which is fitted upon a meter rod. We may now proceed either by the "card" method or by the "pin" method.

The card method. Mount a card upon a cork slider on one side of the lens, and a candle upon another cork slider on the other side of the

lens. Light the candle, and darken the room. Place the candle at a distance from the lens greater than its focul length ; this distance is the value of u. Move the card till a distinct image of the candle flame is obtained. Measure the distance of the card from the lens ; this distance is the value of v. Then compare the observed value of v with that calculated from the formula.

The pin method. Substitute a pin for the card. Place one pin at a distance from the lens greater than its focal length ; this distance is u. Move the other pin till you find by the method of sighting that it coincides with the image of the first pin. Then measure v, and compare with the computed value of v.

If the focal length of the lens is large, it may be necessary to use in place of one meter rod two rods placed end to end.

4. *Construct by points the real image of a straight arrow formed by a convex lens.*

Draw an arrow, on a long sheet of paper, near one end. Place the lens perpendicular to the paper at a distance from the arrow of about $1\frac{1}{2}$ times the focal length. Stick a pin upright into the tip of the arrow, and by means of another pin on the other side of the lens, determine the position of the image of the first pin by the method of sighting already explained. In this way find several points corresponding to images of the pin when stuck into different parts of the arrow. Unless the lens is very thin, the image points will not lie in a straight line ; they will lie in a curved line which is concave towards the lens.

5. *Construct by points the virtual image of a straight arrow formed by a convex lens.*

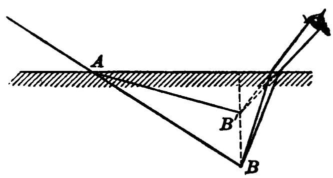
Place in a vertical position near the edge of a long sheet of paper the lens, and behind it, but parallel to it, draw the arrow. Stick a pin upright into the tip of the arrow. Look *through* the lens at this pin (what you really see is the virtual image of the pin), and look *over* the lens at another pin, which you stick into the paper in line with the first pin, but at a greater distance from the lens. Adjust the second pin till, on moving the head from side to side, the image and the pin move together. In like manner obtain image points for other points of the arrow. The line joining the points obtained will be a curved line with its convex side towards the lens.

Class-room Exercises. Page 402.

1. *Construct to scale the path of a ray of light through a thick plate of crown glass. Index of refraction* $\frac{3}{2}$.

The construction is in all respects similar to that in Fig. 319, only OD must be taken so that $OD : OA = 2 : 3$. The proof that the construction is correct depends on the similarity of the triangles AOC and DOE, and the fact that $FB = OE$.

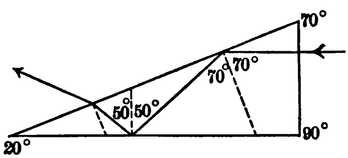
2. *Show by a diagram how a straight stick held obliquely and immersed partly in water appears to a person in the air, and trace a pencil of rays from the lower end of the stick to the eye.*



The stick will appear bent at the point where it enters the water, as shown in the figure.

The rays of light which proceed from the end B of the stick to an eye at E are refracted when they leave the water, so that they appear to the observer to proceed from a point B' nearer the surface of the water. The other points of the stick between A and B are similarly raised ; so that the immersed portion of the stick appears to have the position AB'.

3. *The angles of a glass prism are* 90°, 70°, *and* 20°. *A ray of light enters the prism normally at the face bounded by the angles* 90° *and* 70°. *The critical angle for the glass is* 41°. *Prove that the ray will suffer two internal reflections before it leaves the prism.*



The path of the ray through the glass and the angles of incidence and reflection are shown in the figure.

4. *The focal length of a convex lens is* 20 *cm. Find the positions of the images of a small object if the distances of the object from the lens are* 20 *meters,* 40 *cm.,* 15*cm., and* 5 *cm., respectively. Are the images real or virtual?*

From the formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we have $v = \frac{uf}{u-f}$.

If $u = 20$ meters $= 2000$ cm., $v = 20.2$ cm. Image real.
If $u = 40$ cm., $v = 40$ cm. Image real.
If $u = 15$ cm., $v = -60$ cm. Image virtual.
If $u = 5$ cm., $v = -6.66$ cm. Image virtual.

5. *If the distance of an object from a convex lens is twice the focal length of the lens, prove that the image is at the same distance on the other side of the lens.*

If in the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we put $u=2f$, we have

$$v=\frac{uf}{u-f}=\frac{2f^2}{f}=2f.$$

6. *An object is 60 cm. from a lens on one side and the image is 15 cm. on the other side. What is the focal length?*

Here $u=60$, $v=15$. By substitution in the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we have $f=12$ cm.

7. *An object is 12 cm. from a lens, and its image is 72 cm. from the lens on the same side. Is the lens convex or concave? What is its focal length? Illustrate by a diagram.*

The image, being on the same side of the lens as the object, must be virtual. Since it is farther from the lens than the object, the lens must be convex (see Figs. 334 and 335).

$u=12$, $v=-72$; therefore

$$-\frac{1}{72}+\frac{1}{12}=\frac{1}{f}$$

$$\frac{5}{72}=\frac{1}{f}$$

$$f=14.4 \text{ cm.}$$

8. *An object is 36 cm. from a lens, and its image is 3 cm. from the lens on the same side. Is the lens convex or concave? What is its focal length? Illustrate by a diagram.*

The image is virtual, and the lens is concave.

$u=36$, $v=3$;

Substituting in the formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$,

We have $\frac{1}{3}-\frac{1}{36}=\frac{1}{f}$.

$$\frac{11}{36}=\frac{1}{f}$$

$f=3.3$ cm., nearly.

9. *A small object moves along the axis of a concave lens towards the lens. Trace the changes in the position and size of the image. Is the image real or virtual?*

When the object is very far from the concave lens, the image is formed sensibly at the principal focus. While the object moves up to the lens, the image moves from the principal focus up to the lens. The image is always virtual.

10. *A candle is 8 feet from a wall. When a lens is held 2 feet from*

the candle a distinct image is thrown on the wall. Find the focal length of the lens. Compare the image and the object in respect to size.

Here $u = 2$ ft., $v = 6$ ft.; therefore $\frac{1}{2} + \frac{1}{6} = \frac{1}{f}$, whence $f = 1\frac{1}{2}$ ft. $= 18$ inches.

The image is 3 times as long as the object.

11. *At what distance from a convex lens must an object be placed in order that the image may be half as large as the object? Focal length of lens = 30 cm.*

Since the image is half as large as the object, $v = \frac{u}{2}$.

Therefore $\frac{2}{u} + \frac{1}{u} = \frac{1}{30}$, $\frac{3}{u} = \frac{1}{30}$, $u = 90$ cm.

12. *A convex lens of 6 inches focal length is used to read the graduations on a scale. How far must it be held from the scale to magnify them three times?*

To magnify 3 times, the image must be 3 times as far from the lens as the object is, or $v = 3u$. Also, since the image is virtual, v is negative.

Therefore $-\frac{1}{3u} + \frac{1}{u} = \frac{1}{6}$, whence $u = 4$ inches.

13. *An arrow 5 cm. long placed 10 cm. from a lens at right angles to the axis has a virtual image 30 cm. from the lens. If the arrow were placed 30 cm. from the lens, how far from the lens would the image be?*

Substituting $u = 10$, $v = 30$ in the formula,

$$\frac{1}{u} - \frac{1}{v} = \frac{1}{f}$$

we obtain $f = 15$ cm.

If the arrow is placed 30 cm. from the lens, the image will be real, and on the opposite side of the lens. Substituting in the formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

we obtain $v = 30$ cm.

Review Exercises on Chapter VIII. Page 427.

1. *What is the cause of sound? What property of matter makes sound possible? Give reasons for your answers.*

See § 333.

2. *How would you show that sound cannot travel through a vacuum?*

See § 333.

3. *Describe transverse and longitudinal vibrations, illustrating by diagrams. Define period and frequency.*

See § 334.

4. *What law of force will cause simple harmonic vibrations? What relation exists between such vibrations and uniform circular motion?*

See § 335.

5. *Explain the meaning of the formula $v = ln$.*

See § 337.

6. *What is the velocity of sound in air? How is it affected by temperature?*

See § 339.

7. *Explain echoes.*

See § 341.

8. *What is the distinction between noise and musical sound?*

See § 342.

9. *On what does the loudness of a sound depend?*

See § 343.

10. *On what does the pitch of a sound depend?*

See § 344.

11. *Define a musical interval and an octave.*

See § 346.

12. *What are the laws of the vibration of strings?*

See § 347.

13. *Define, by taking the case of a vibrating string, harmonic overtones, nodes, antinodes, and stationary waves.*

See § 348.

14. *Compare the vibrations of open and closed organ pipes.*

See § 349.

15. *What is the law of the composition of vibrations?*

See § 350.

16. *What are sympathetic vibrations? Give examples.*

See § 351.

17. *Define and illustrate resonance.*

See § 352.

18. *Give an example of the interference of sound.*

See § 353.

19. *What are beats? How is their number in any case determined?*

See § 354.

20. *Describe Helmholtz's resonators and how he used them.*

See § 357.

21. *Illustrate what is meant by the quality of a musical sound. How are different qualities of sound explained?*

See § 358.

22. *What is the physical explanation of harmony and of discord?*
See § 359.

23. *Explain by a diagram the formation of a shadow, and distinguish between the umbra and the penumbra.*
See § 364.

24. *What is the velocity of light? Describe one method by which it has been determined.*
See § 365.

25. *Prove that the illuminating powers of two sources of light are proportional to the squares of their distances from a surface which they illuminate with equal intensity.*
See § 366.

26. *Prove that the image of a luminous point in front of a plane mirror is on the normal from the point to the mirror, and as far behind the mirror as the point is before it.*
See § 371.

27. *An object is placed midway between two parallel mirrors. Show that a series of images arranged on a straight line will be formed. Illustrate by a diagram how the series of images are formed.*
See § 373.

28. *Define the terms principal focus and focal length.*
See § 375.

29. *Prove the mirror formula. What are conjugate foci?*
See § 378.

30. *Construct the image of an object placed before a concave mirror beyond the principal focus.*
See § 379.

31. *Construct the image of an object placed between a concave mirror and the principal focus.*
See § 379.

32. *What are the laws of refraction? Define the index of refraction.*
See § 381.

33. *Explain why a stick appears bent where it enters the surface of water. Illustrate by a diagram.*
See solution of Ex. 2, p. 402.

34. *Explain, with a diagram, the total reflection of light. Define the critical angle. What is its value for water? For glass?*
See § 383.

35. *Explain by a diagram the passage of light through a window pane.*
See § 384.

36. *Draw a diagram showing the passage of light through a prism. What is the best way to change the direction of rays of light by 90°?*
See § 385.

37. *How do convex and concave lenses differ in their action on light?*
See § 386.

38. *Define the optical center of a lens.* See § 388.

39. *Construct the image of an object placed between a convex lens and the principal focus.* See § 390.

40. *What is spherical aberration? What are its effects? How is spherical aberration prevented?*
See § 391.

41. *How did Newton analyze white light?* See § 392.

42. *How can white light be produced by synthesis?* See § 393.

43. *Give a general explanation of the rainbow.* See § 395.

44. *What is chromatic aberration? How is it prevented?* See § 396.

45. *Describe briefly the spectroscope and its uses.* See §§ 398, 402.

46. *Explain the Fraunhofer lines in the solar spectrum.* See § 401.

47. *Why can we see distinctly objects at very different distances?*
See § 404.

48. *What are the advantages of vision with two eyes?* See § 408.

49. *Give an example of an after-image. How do you account for it?*
See § 409.

50. *Describe, with diagram, the compound microscope.* See § 412.

51. *Describe, with diagram, the astronomical telescope.* See § 414.

52. *Give a brief account of the two theories about light, and reasons for preferring the wave theory.*
See § 417.

53. *How are the colors of soap bubbles explained?* See § 418.

54. *What is the physical cause of color? Give examples of pure colors, mixed colors, and complementary colors.*
See §§ 423, 424.

55. *In a room illuminated with sodium light all objects would appear either yellow or black. Explain.*
See §§ 424, 425.

TABLES.

HEAT TABLES.

SUBSTANCE.	COEFFICIENT LINEAR EXPANSION 0° to 100° C.	MELTING POINT C° (Approximately.)	MEAN SPECIFIC HEAT IN CALORIES.	LATENT HEAT OF FUSION IN CALORIES.
Gold	0.000014	1150°	0.032	
Silver . . .	0.000019	1000°	0.056	21.1
Platinum . .	0.000009	2000°	0.032	27.2
Iron, cast . .	0.000011	1100°	0.113	23.
Iron, wrought	0.000012	1600°	0.108	
Steel. . . .	0.000012	1400°	0.118	
Copper . . .	0.000017	1100°	0.093	
Zinc	0.000029	400°	0.094	28.
Lead. . . .	0.000027	330°	0.031	5.5
Tin	0.000023	230°	0.056	13.
Brass . . .	0.000019	1000°	0.093	
Aluminium .	0.000023	700°	0.219	
Glass . . .	0.000009	400°	0.190	
Ice		0°	0.504	

Water, coefficient cubical expansion (0° to 100°), 0.00043; latent heat of fusion, 80 calories; latent heat of vaporization, 536 calories.

Alcohol, coefficient cubical expansion (0° to 100°), 0.00104; freezing point, —130°; boiling point, 78°; specific heat, 0.612 calories; latent heat of vaporization, 202 calories.

Mercury, coefficient cubical expansion (0° to 100°), 0.00018; freezing point, —39°; boiling point, 357°; specific heat, 0.033 calories; latent heat of fusion, 2.82 calories.

Dry Air, specific heat under constant pressure, 0.2375 calories; sp. heat under constant volume, 0.1685 calories.

Steam, specific heat under constant pressure, 0.3787; specific heat under constant volume, 0.2646.

BOILING POINT OF WATER UNDER DIFFERENT PRESSURES.

Boiling Point, C°.	Pressure in cm. of Mercury.	Boiling Point, C°.	Pressure in cm. of Mercury.	Boiling Point, C°.	Pressure in cm. of Mercury.
90°	52.5	100°	76.0	150°	358.1
91°	54.6	105°	90.6	155°	408.9
92°	56.7	110°	107.5	160°	465.2
93°	58.8	115°	126.9	165°	527.5
94°	61.1	120°	149.1	170°	596.2
95°	63.4	125°	174.4	175°	671.7
96°	65.8	130°	203.0	180°	754.7
97°	68.2	135°	235.4	185°	845.3
98°	70.7	140°	271.8	190°	944.2
99°	73.3	145°	312.6	195°	1052.0
100°	76.0	150°	358.1	200°	1168.9

SPECIFIC GRAVITIES.

Solids.

Gold	19.3	Sulphur, roll	2.0
Silver	10.5	Sand, dry	1.4
Platinum	21.5	Clay	1.9
Iron, cast	7.2	Brick	1.8
Iron, wrought	7.8	Marble	2.7
Steel	7.8	Glass, crown	2.6
Copper	8.9	Glass, flint	3.5
Zinc	7.1	Porcelain	2.4
Lead	11.4	Slate	2.7
Tin	7.3	Cork	0.24
Nickel	8.9	Ice	0.92
Aluminium	2.6	White Pine, dry	0.42
Brass	8.4	White Oak, dry	0.86
Carbon, diamond	3.5	Maple, dry	0.64
Carbon, graphite	2.3	Ash, dry	0.77
Carbon, gas carbon	1.9	Hickory, dry	1.00
Carbon, wood charcoal	1.6	Ebony, dry	1.1

Liquids at 0° C.

Sea Water	1.026	Sulphuric Acid, conct.	1.84
Alcohol	0.8	Nitric Acid, conct.	1.55
Chloroform	1.5	Hydrochloric Acid, conct.	1.27
Ether	0.73	Milk	1.03
Glycerine	1.27	Linseed Oil	0.94
Mercury	13.6	Kerosene	0.79

Gases at 0° C. and 76 cm.

Referred to dry air at 0° C. and 76 cm.

	Specific Gravity.	Weight of One Liter in Grams.		Specific Gravity.	Weight of One Liter in Grams.
Dry air	1.	1.293	Hydrogen	0.069	0.0896
Oxygen	1.105	1.43	Carbon dioxide,	1.52	1.965
Nitrogen	0.971	1.25	Ammonia	0.59	0.762

VALUES OF BROWN AND SHARPE WIRE GAUGE NUMBERS.

No.	Diameter in mm.	No.	Diameter in mm.
1	7.348	17	1.150
2	6.544	18	1.024
3	5.827	19	0.912
4	5.189	20	0.812
5	4.621	21	0.723
6	4.115	22	0.644
7	3.656	23	0.573
8	3.264	24	0.511
9	2.906	25	0.455
10	2.582	26	0.405
11	2.305	27	0.361
12	2.053	28	0.321
13	1.828	29	0.286
14	1.628	30	0.255
15	1.459	31	0.227
16	1.291	32	0.202

ELECTRIC RESISTANCE.

K denotes the resistance in ohms of a wire 1 kilometer long and 1 square millimeter in cross-section.

Substance.	Value of K.	Substance.	Value of K.
Aluminium	30	Zinc	59
Silver	16	Lead	196
Platinum	135	Tin	105
Iron, cast	943	Brass	82
Iron, wrought	270	German silver	236
Steel	190	Graphite, Siberian	11500
Copper	17	Graphite, retort	49300